

A Bose–Einstein condensate analogue of Misner space: a design study of the kinematic image-sum mechanism

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Abstract

Hawking’s chronology protection conjecture attributes the absence of time machines to the divergence of vacuum polarization at a chronology horizon. Misner space—two-dimensional Minkowski spacetime quotiented by a boost—is the cleanest chronology-horizon geometry: it is locally flat, and its horizon is a compactly generated null surface on which the boost holonomy piles up field modes. We present a *design study*, not a completed experiment, for realizing the Milne quotient in a quasi-one-dimensional ^{87}Rb Bose–Einstein condensate confined to a ring whose radius collapses linearly in time, $R(t) = R_0 - Vt$ with $V = \lambda c_s/(2\pi)$. In the acoustic regime the azimuthal phonons propagate on the Milne metric $ds^2 = -dT^2 + T^2 d\psi^2$ with $\psi \sim \psi + \lambda$. The ring realizes the quotient cylinder: in the covering space the boost orbit is a hyperbola, and the ring is the quotient circle obtained by identifying points along it, not the orbit itself. The ring’s $R = 0$ limit is a conical tip, not a compactly generated null horizon, so the construction is a kinematic analogue of the image-sum geometry and does not test the Kay–Radzikowski–Wald theorem in any dimension. The measured quantities are the equal-time density correlation $G(\Delta\phi)$ on the ring and the angular-mode occupations. The stress coefficient $S_\lambda = 2 \sum_{n \geq 1} q^n / (1 - q^n)^2$ ($q = e^{-\lambda}$) is *inferred* from the short-distance (coincidence-limit) scaling of G ; it is not the amplitude of the finite-separation density correlation, and the two must not be conflated. We give corrected condensate parameters ($c_s = 3.263$ mm/s, not 4.62; $V = 0.519$ mm/s and a 96 ms sweep at $\lambda = 1$), recompute the horizon temperature (25.1 nK), the Bragg scale $c_s/(2\pi\xi) = 3.28$ kHz, and the thermal coherence length, and report the feasibility verdict. The antipodal correlation sits at $\Delta\psi = \lambda/2$ (an arc $\pi R_0 = 157$ μm) and requires $T \lesssim 0.16$ nK; the first image is the $n = \pm 1$ winding term at $\Delta\psi = \lambda$, i.e. at the full ring $2\pi R_0 = 314$ μm (the same point), and requires $T \lesssim 0.08$ nK. Bragg modes at $k\xi \sim 2.5$ have a Beliaev lifetime $\tau \sim 50$ μs , and a fixed- q Bragg scan resolves discrete Bogoliubov peaks, not a log-periodic frequency comb. The 50 μm correlation and fixed- q Bragg channels are therefore not feasible at the stated parameters; the time-of-flight angular-mode (n_m) channel is not yet assessed, and the final ring has only $N(R_{\min}) \approx 2.1 \times 10^2$ atoms and ten azimuthal modes. We state the open blockers explicitly—the image identification and its temperature requirement (B3), Beliaev damping (B4), the fixed- q Bragg comb (B5), and $N(R_{\min})$ (B6)—and define the measurements and theory needed to close them. The study is complementary to the 2026 optical analogue of Bermudez and Leonhardt. No experiment described here has been performed.

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1 Introduction

1.1 The physical core of chronology protection

Hawking’s chronology protection conjecture proposes that the laws of physics prevent the appearance of closed timelike curves (CTCs) [2]. The proposed enforcement mechanism is semiclassical. As

a spacetime is deformed toward CTC formation, a *chronology horizon*—the boundary of the region containing CTCs—forms before any CTC exists. Null geodesics winding around the horizon pile up, virtual particles acquire a boost holonomy at each circuit, and the renormalized stress-energy tensor $\langle T_{\mu\nu} \rangle$ of a quantum field diverges. Fed back through the semiclassical Einstein equations, the divergence is supposed to destroy the horizon before CTCs can form [2].

The rigorous core of this picture is the Kay–Radzikowski–Wald (KRW) theorem [3]. For a *compactly generated* Cauchy horizon—one whose null generators all remain in a compact region—no quantum state can be Hadamard everywhere: the two-point function fails to be Hadamard at the horizon, and $\langle T_{\mu\nu} \rangle$ is there singular or undefinable. Explicit computations in Taub–NUT space [4], wormhole time machines [5], and Misner space [2, 7, 6] support the same conclusion: vacuum polarization diverges as the horizon is approached.

Every step from these results to “time machines are impossible” crosses a known crack [12]. The two cracks relevant here are:

1. *Scope.* KRW applies to compactly generated horizons. It is silent on non-compactly generated machines—rotating cylinders, Krasnikov-type constructions, and the compact vacuum cores of Ori [13] and Xavier [19]. The Ori and Xavier cores are the leading constructive candidates, and their horizons are not covered by the theorem.
2. *State choice.* For Misner space, self-consistent Hadamard states with $\langle T_{\mu\nu} \rangle = 0$ everywhere off the horizon have been constructed [8, 9, 10]. The reply is that on the horizon itself the operator remains undefinable [11]; but the physical question—which state does a given formation process prepare?—is not settled by the distributional argument. Quantum inequalities bound how negative the stress can be [29, 30]; the state-choice question is whether the quotient stress can be kept finite without violating them.

Quantum mechanics itself does not forbid CTCs: Hartle’s generalized quantum mechanics in nonchronal spacetimes is internally consistent [28]. Any chronology protection must therefore come from the dynamics of gravity plus quantum fields, not from quantum logic. Holography has strengthened the conjecture in a different form: every controlled quantum-gravity construction of a traversable wormhole is causally safe [26, 27]. Holographic models also show that backreaction can turn a chronology horizon into a spacelike singularity [15], and the holographic time-machine construction makes the mechanism precise [14]. Chronology protection is dead as a theorem and alive as an emergent pattern. To our knowledge, no laboratory measurement of the image-sum mechanism has been reported; the 2026 optical work discussed below is a proposal [20].

1.2 Why an analogue experiment?

Analogue gravity reproduces the *kinematics* of quantum fields on a curved background, not the dynamics of the background [22]. A Bose–Einstein condensate (BEC) with a space- or time-dependent flow has phonons obeying a wave equation on an effective acoustic metric; acoustic horizons and their Hawking radiation have been measured [25, 24]. The same platforms can, in principle, realize the kinematics of a chronology horizon. Barceló, Eguía Sánchez, García-Moreno and Jannes have shown that an analogue system can simulate spacetimes with CTCs only when this does not require a chronology horizon separating CTC from non-CTC regions, and they identify a universal near-horizon behaviour that plays the role of an analogue chronology protection [21]. The obstruction is not a prohibition on experiment: it defines the accessible regime. An analogue experiment can approach the would-be horizon from the chrontal side, where the divergence mechanism is predicted to operate, without ever entering the nonchronal region.

The closest experimental comparator is the 2026 optical proposal of Bermudez and Leonhardt [20]. In their scheme time travel is inherently related to lasing, and amplified vacuum fluctuations above laser threshold establish the quantum limits. The BEC design differs in three ways that matter for a measurement: the phonon field is a genuine many-body quantum field with a known ultraviolet completion (the healing length), the initial state can be prepared and varied, and the observable is a density correlation rather than an optical amplification threshold. The two studies are complementary, not competing; they probe different error budgets of the same conjecture.

1.3 The Misner quotient as the cleanest target

Misner space is the quotient of two-dimensional Minkowski spacetime by a boost [1, 2]. It is locally flat, so its quantum-field theory is exactly solvable in the same sense as Minkowski space; for Misner space itself the chronology horizon is the paradigm compactly generated horizon; and its image sum is the simplest explicit realization of Hawking’s pile-up. It is therefore the natural target for a kinematic analogue. If the image-sum mechanism fails to appear in the cleanest quotient geometry, the negative result is informative; if it appears, the curved Ori and Xavier cores become the interesting exceptions. We emphasize at the outset, however, that the ring construction of Section 2 realizes the quotient *cylinder* and its conical tip, not the null horizon of the full Misner space; the global compactly generated setting is not reproduced.

The 4D Misner geometry is, in a model-dependent sense, the $f = 0$ limit of the Ori core: the local covering space of the Ori 2005 compact vacuum core is a plane-fronted wave, and the Misner quotient is its flat, maximally symmetric corner. The companion image-series work (in preparation) reports that the stress on the Ori core follows the Misner ϵ^{-2} law but is capped near the horizon locus; the Xavier core has a degenerate horizon family with a structurally different winding behaviour [19, 32, 33]. Those companion results are model-dependent and are cited here only qualitatively; the present design study does not rely on their numerical values (Section 7.2).

1.4 Scope and summary

This paper is a design study. Section 2 constructs the analogue geometry: the Misner quotient, the phonon effective metric, the collapsing-ring implementation, and the precise sense in which the ring realizes the quotient cylinder. Section 3 defines what is measured—the equal-time density correlation and the angular-mode occupations—and separates it from what is inferred, the image-sum coefficient S_λ . Section 4 gives the corrected condensate parameters, the collapse protocol, the detection, and the shot budget. Section 5 is the systematic budget. Section 6 states the feasibility verdict and the blockers. Section 7 relates the study to the optical analogue and, qualitatively, to the Ori/Xavier cores. Section 8 lists the milestones, and Section 9 concludes.

The claim is deliberately narrow: the design target is to determine whether the kinematic image-sum structure of the Milne quotient is realized in a system whose effective geometry is the quotient cylinder. It does not create a CTC, does not realize a null chronology horizon, does not test gravitational backreaction, and does not refute or prove chronology protection. The nominal parameters of this preprint do not reach the target; the design study exists to make the gap explicit.

Table 1: The Misner quotient. The table records the geometric objects used in the text; the horizon is approached from the chrontal side ($T > 0$).

Covering space	$\mathbb{R}^{1,1}, ds^2 = -dU dV$
Identification	$U \sim e^{-\lambda}U, V \sim e^{\lambda}V$
Milne coordinates	$U = Te^{\psi}, V = Te^{-\psi}$
Quotient metric	$ds^2 = -dT^2 + T^2 d\psi^2, \psi \sim \psi + \lambda$
Horizon	$T = 0$ ($U = 0$ or $V = 0$)
CTC region	$UV < 0$ ($T^2 < 0$), not accessed
Holonomy per circuit	$e^{\pm\lambda}$
Circle circumference	λT

2 The analogue geometry

2.1 Misner space and the boost quotient

Let the covering space be two-dimensional Minkowski spacetime with null coordinates

$$U = t - x, \quad V = t + x, \quad ds^2 = -dU dV. \quad (1)$$

A boost of rapidity λ acts as

$$B_\lambda : \quad U \mapsto e^{-\lambda}U, \quad V \mapsto e^{\lambda}V. \quad (2)$$

Misner space is the quotient

$$\mathcal{M}_\lambda = \mathbb{R}^{1,1} / \langle B_\lambda \rangle. \quad (3)$$

In the future wedge $U, V > 0$ introduce

$$U = Te^{\psi}, \quad V = Te^{-\psi}, \quad T = \sqrt{UV} > 0, \quad \psi = \frac{1}{2} \ln(V/U). \quad (4)$$

The metric becomes the Milne metric

$$ds^2 = -dT^2 + T^2 d\psi^2, \quad (5)$$

and the boost acts as a translation $\psi \mapsto \psi - \lambda$. The quotient therefore identifies

$$\psi \sim \psi + \lambda. \quad (6)$$

The circumference of the ψ -circle at proper time T is λT . The Killing vector ∂_ψ has norm squared T^2 : it is spacelike in the Milne wedge, null at $T = 0$, and timelike in the region $UV < 0$. The surface $T = 0$ is the chronology horizon; the region $UV < 0$ is the CTC region. The horizon is generated by closed null geodesics (the “fountain”), and the boost holonomy per circuit is $e^{\pm\lambda}$.

Equation (5) is locally flat: the Riemann tensor vanishes identically. All of the chronology physics is in the identification (6), not in curvature. This is why the geometry is the natural target for an analogue system: a condensate that reproduces the identification reproduces the local quotient kinematics. Whether it also reproduces the global horizon is a separate question, addressed below.

2.2 Phonons in a condensate

Consider a three-dimensional BEC of N atoms of mass m , with density n , s -wave scattering length a_s , and healing length

$$\xi = \frac{1}{\sqrt{8\pi n a_s}}. \quad (7)$$

The speed of sound is the thermodynamic (Bogoliubov) sound speed,

$$c_s = \sqrt{\frac{gn}{m}} = \frac{\hbar}{\sqrt{2}m\xi} = \sqrt{\frac{4\pi\hbar^2 a_s n}{m^2}}, \quad g = \frac{4\pi\hbar^2 a_s}{m}. \quad (8)$$

The first and third forms are identical; the equivalent healing-length form is $c_s = \hbar/(\sqrt{2}m\xi)$. Since $\hbar/(m\xi) = \sqrt{8\pi}\hbar\sqrt{n a_s}/m = \sqrt{2}c_s$, the common shorthand $c_s = \hbar/(m\xi)$ is too large by $\sqrt{2}$. The numerical consequences are collected in Table 4. For wavelengths long compared with ξ , the phase fluctuation φ (the phonon field) obeys the massless wave equation on the acoustic metric [22]

$$ds_{\text{ac}}^2 = \frac{n}{mc_s} [-(c_s^2 - v^2)dt^2 - 2v dt dx + dx^2], \quad (9)$$

where v is the local flow velocity. In two spacetime dimensions the massless wave equation is conformally invariant, so the prefactor $n/(mc_s)$ drops out of the phonon dynamics. This is the technical reason the design is clean: in the acoustic regime the phonons see the *conformal class* of the metric, and the Misner quotient is conformally flat.

For a ring of radius $R(t)$ with no azimuthal flow, the azimuthal phonons see the effective metric

$$ds_{\text{ring}}^2 \propto -c_s^2 dt^2 + R(t)^2 d\phi^2, \quad \phi \sim \phi + 2\pi. \quad (10)$$

The radial flow that drives the collapse is assumed uniform; its leading effect is a calculable renormalization of c_s and of the effective ring radius, included in the M1 Gross–Pitaevskii simulation. The azimuthal sector is decoupled from the radial flow at leading order because the two directions are orthogonal. Set $T = c_s(t_0 - t)$, the proper time remaining before the ring reaches $R = 0$, and

$$R(T) = \frac{\lambda}{2\pi} T. \quad (11)$$

Then with $\psi = (\lambda/2\pi)\phi$,

$$ds_{\text{ring}}^2 \propto -dT^2 + T^2 d\psi^2, \quad \psi \sim \psi + \lambda. \quad (12)$$

Equation (12) is locally the Misner metric (5). The ring at radius $R(T)$ is the quotient circle of the boost orbit: the orbit in the covering space is the hyperbola $UV = T^2$ at fixed T , and the ring is its quotient by the boost, with circumference $\lambda T = 2\pi R(T)$. The angular coordinate is the quotient direction. It is not correct to call the ring itself the orbit, and the distinction matters at $R = 0$ (below).

The identification (6) is thus not imposed by an external clock. It is the periodic boundary condition of the ring, read in Milne coordinates. The boost parameter is fixed by the collapse rate,

$$V = -\frac{dR}{dt}, \quad \lambda = \frac{2\pi V}{c_s}. \quad (13)$$

A ring that collapses at a fraction $\lambda/(2\pi)$ of the sound speed realizes the Milne quotient cylinder with rapidity λ . The “identification drive” of the experimental design is therefore a controlled radius ramp, not a separate moving lattice.

The $R = 0$ limit is a conical tip. As $T \rightarrow 0$ the quotient circle of circumference λT shrinks to a point. For $\lambda = 2\pi$ the quotient has no deficit and the tip is regular; for $\lambda \neq 2\pi$ it is a conical singularity with deficit $2\pi - \lambda$. The chronology horizon of full Misner space is the null surface $T = 0$ (equivalently $UV = 0$), generated by closed null geodesics that remain at finite affine distance. The ring has no such null surface: its worldvolume is two-dimensional and the angular Killing vector ∂_ϕ is spacelike everywhere the ring exists, becoming degenerate only at the tip. The ring therefore reproduces the local image-sum structure of the Milne wedge and the approach to the shrinking quotient circle, but it does not reproduce a compactly generated null horizon, and it does not realize the CTC region, in which ∂_ϕ would have to become timelike. What would be missing for a chronology-horizon analogue is exactly that: a null surface on which the identification direction becomes null and then timelike, with closed null geodesics of finite affine length. A purely spatial ring cannot supply it.

2.3 The collapsing ring

The experimental protocol is:

1. Prepare a quasi-one-dimensional condensate in a ring of radius R_0 and circumference $2\pi R_0$.
2. Ramp the ring radius linearly in time, $R(t) = R_0 - Vt$ with $V = \lambda c_s/(2\pi)$, so that $R(T) = (\lambda/2\pi)T$ over the accessible range.
3. Stop the ramp at $R_{\min} \simeq 10\xi$, where the acoustic description is still assumed controlled but the shrinking circle is close.
4. Measure the equal-time density correlation and the angular-mode occupations as functions of R and λ .

The center $R = 0$ is the conical tip of the quotient, not a null horizon. The CTC region $T^2 < 0$ has no ring realization: it would require the angular direction to become timelike, which a spatial ring cannot do. The protocol approaches the tip from the chrontal side and never enters the nonchronal region. This is the regime in which the Barceló et al. analysis identifies the universal near-horizon behaviour; the obstruction forbids the CTC side, not the approach [21].

Figure 1 is a schematic of the construction. The ring of radius $R(t)$ is the quotient circle of the boost orbit, the center is the conical tip, and the protocol stops at $R_{\min}$ before the tip. The figure is not to scale: the accessible range is $R_0/R_{\min} \simeq 31.6$.

2.4 What the analogue does and does not reproduce

The analogy is exact only in a limited sense, and it is important to state the limits at the outset.

What is reproduced. In the acoustic regime $k\xi \ll 1$, the azimuthal phonon field is a 2D conformal scalar on the metric (12). The quotient identification, the boost holonomy, the shrinking quotient circle, and the local image-sum mode structure are reproduced. The *kinematics of the quotient* is therefore realized.

What is not reproduced. The BEC is a Galilean many-body system; the boost is an emergent symmetry of the phonon sector, not of the microphysics. The CTC region $T^2 < 0$ is not physical. The ring's $R = 0$ limit is a conical tip, not the compactly generated null horizon of Misner space: the closed null geodesics of the horizon are absent, and the angular direction never becomes null. The condensate does not obey Einstein's equations, and the phonon stress does not backreact gravitationally; it backreacts on the condensate through the Gross-Pitaevskii nonlinearity, which

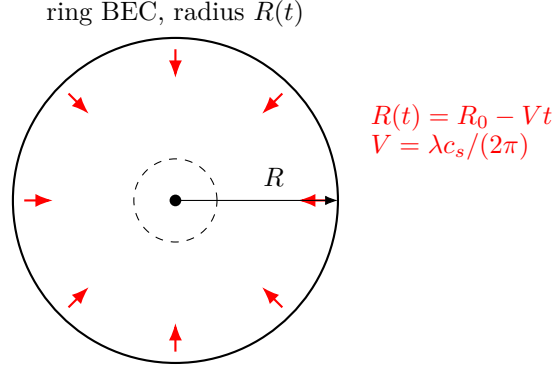


Figure 1: Schematic of the collapsing-ring implementation, not to scale. The ring of radius $R(t)$ is the quotient circle of the boost orbit of the Milne quotient; the center $R = 0$ is the conical tip at which the quotient circle shrinks, not a null chronology horizon; the dashed circle marks $R_{\min} \simeq 10\xi$, where the protocol stops. The approach is from the chronal side. The rapidity is set by the collapse rate, $\lambda = 2\pi V/c_s$.

is a different dynamics. The construction cannot create a time machine and cannot test the KRW theorem in any dimension, because it does not realize the theorem's hypothesis (a compactly generated Cauchy horizon). It can ask whether the local image-sum structure is realized on the quotient cylinder and whether the state prepared by a physical collapse follows the diverging vacuum or an adapted, finite one. That is the claim, and no more.

3 The observable

3.1 The image sum and the mode density

On the quotient, the two-point function is the image sum of the covering-space two-point function:

$$G_\lambda(x, x') = \sum_{n \in \mathbb{Z}} G_0(x, B_\lambda^n x'). \quad (14)$$

The stress tensor is the coincidence limit of derivatives of G_λ . In the full Misner quotient, as x approaches the horizon $T = 0$, the images accumulate and the sum diverges. This is Hawking's pile-up in its simplest form. The ring reproduces only the local part of this construction (Section 2.4); the divergence itself is not reached.

For a 2D conformal scalar, the null-null component of the stress on the Misner quotient can be written as

$$\langle T_{uu} \rangle(u) = \mathcal{N} \hbar c_s \frac{S_\lambda}{u^2} \quad (u \rightarrow 0), \quad S_\lambda = 2 \sum_{n=1}^{\infty} \frac{q^n}{(1 - q^n)^2}, \quad q = e^{-\lambda}, \quad (15)$$

where u is the affine null coordinate measured from the horizon and $\mathcal{N}$ is a dimensionless normalization fixed by the standard point-splitting convention. The $1/u^2$ law and the λ -dependence of S_λ are exact for the conformal scalar; the absolute normalization is the M0 theory task (Section 8). For small λ ,

$$S_\lambda \simeq \frac{\pi^2}{3\lambda^2} \quad (\lambda \ll 1), \quad (16)$$

Table 2: Dimensional map of the pile-up law. The first row is the present design; the others are future extensions. The exponents are exact for a conformal scalar; the coefficients $S_\lambda^{(d)}$ for $d \geq 2$ are not computed here.

Spacetime	Field	Predicted stress	Platform
1+1	2D conformal scalar	$\propto S_\lambda/u^2$	quasi-1D ring (this design)
2+1	3D scalar	$\propto S_\lambda^{(2)}/u^3$	quasi-2D gas (future)
3+1	4D scalar	$\propto S_\lambda^{(3)}/u^4$	3D gas (future)

and for $\lambda = 2\pi$ (the smooth-cone, Rindler-like anchor) $S_\lambda = 3.756 \times 10^{-3}$. The function is monotonically decreasing in λ : a slower collapse (smaller λ) produces a larger coefficient because the images are closer in the covering space.

Equation (15) is the quantitative core of the image-sum prediction. It is important to be precise about what S_λ is: Eq. (15) defines it as the coefficient of the coincidence limit of $\langle T_{uu} \rangle$, i.e. of a short-distance (UV) object. It is *not* the amplitude of the equal-time density correlation at finite separation, and it is not directly measured. Section 3.3 separates the measured correlation from the inferred coefficient. The analysis does not need the absolute normalization to test the shape: the λ -dependence of S_λ and the exponent are parameter-free predictions of the quotient. The normalization can be fixed against the $\lambda = 2\pi$ Rindler limit, where the quotient is smooth and the stress is the standard 2D Unruh stress; that anchoring is the M0 theory task.

In $d + 1$ spacetime dimensions the same image sum gives

$$\langle T \rangle \sim \frac{S_\lambda^{(d)}}{u^{d+1}}, \quad S_\lambda^{(d)} \text{ the } d\text{-dimensional image coefficient.} \quad (17)$$

The exponent is $d + 1$: u^{-2} in 2D, u^{-3} in 3D, u^{-4} in 4D. The 4D case is the $d = 3$ member. The present design is quasi-one-dimensional and would test $d = 1$; a quasi-two-dimensional gas would test $d = 2$; a 3D condensate would test the 4D law. The dimensional dependence is itself a check on the mechanism: a measured exponent that tracks $d + 1$ would be evidence that the image sum, rather than a generic heating effect, is being observed.

3.2 The spectral function and its obstruction

The stress is a UV-sensitive coincidence limit. A tempting spectroscopic observable is the spectral function

$$\rho_\lambda(\omega) = \int dt e^{i\omega t} \langle [\varphi(t), \varphi(0)] \rangle. \quad (18)$$

On the quotient, one might expect the image sum to turn into a geometric rescaling of frequencies,

$$\rho_\lambda(\omega) \stackrel{?}{=} \sum_{n \in \mathbb{Z}} e^{n\lambda} \rho_0(e^{n\lambda} \omega), \quad (19)$$

which would be log-periodic in $\ln \omega$ with period λ . This relation is *not derived* for the BEC ring—the factor $e^{n\lambda}$ is asserted, not obtained from the identification $\psi \sim \psi + \lambda$ —and two obstructions prevent the fixed- q Bragg scan of the original draft from measuring it as described.

Discreteness. On a finite ring the angular momenta are discrete, $m \in \mathbb{Z}$, and the identification acts on the mode label. A Bragg measurement at fixed momentum transfer q probes $S(q, \omega)$, whose peaks are the discrete Bogoliubov frequencies of the populated modes. Scanning ω at fixed q therefore resolves individual Bogoliubov peaks, not a continuum frequency comb; the image

structure is sampled in mode number, not in frequency. At $k\xi \sim 2.5$ the Bogoliubov frequency is $f \simeq 16.7$ kHz, outside the 0.5–10 kHz band quoted in the original draft.

Damping. The probed modes are damped by Beliaev processes even at $T = 0$ (Section 5.3). At $k\xi \sim 2.5$ the standard Beliaev estimate gives $\Gamma \sim 10^4\text{--}10^5$ s $^{-1}$, $\tau \sim 50$ μ s, and a linewidth $\Gamma/2\pi$ of a few kHz, comparable to the proposed 1 kHz resolution and far shorter than the 15–96 ms sweep.

We therefore withdraw the log-periodic spectral comb as an observable. What remains measurable in the UV-adjacent channel is the angular-mode density n_m (the mode occupations) from time of flight, which tests how the collapse populates the quotient modes. Whether that mode density carries the image-sum coefficient is an M0 question, not a result of this study. The healing length still provides the UV cutoff $\omega_{\max} \sim c_s/\xi = 3.28$ kHz and the image count

$$n_{\max} \sim \frac{1}{\lambda} \ln\left(\frac{c_s}{\xi \omega}\right). \quad (20)$$

3.3 The density correlation: measured versus inferred

The primary measured quantity is the equal-time density correlation on the ring,

$$G(\Delta\phi) = \langle \delta n(\phi) \delta n(\phi + \Delta\phi) \rangle - \langle \delta n \rangle^2. \quad (21)$$

For a 1D condensate in the Luttinger-liquid regime, G is a power law in the separation at short distances. On the quotient the two-point function is the image sum (14). The image sum makes G a λ -periodic function of $\Delta\psi$ —the ring periodicity, equivalently period 2π in $\Delta\phi$ —and it is the shape of G within one period, not a new frequency comb, that carries the λ -dependence.

It is essential to separate what is measured from what is inferred:

- *Measured:* $G(\Delta\phi)$ for $0 < \Delta\phi < \pi$, its power-law envelope at short separation, and its period- λ modulation; the angular-mode occupations n_m from time of flight.
- *Inferred:* the stress coefficient S_λ of Eq. (15). It is obtained from the short-distance (coincidence-limit) behaviour of G and the mode density, using the conformal relation between density fluctuations and T_{uu} . The overall normalization is the M0 task; without it only the shape and the exponent are predicted.
- *Not measured, and not predicted here:* the finite-separation density correlation is *not* S_λ . Its amplitude is a different functional of λ and $\Delta\psi$, and no coefficient for it is derived in this paper. The original draft’s claim that G acquires S_λ is therefore withdrawn. Computing the correct finite-separation image sum is part of M0.

Where the images lie. The image sum (14) places the n -th image at $\Delta\psi = n\lambda$ (modulo the period). Since $\psi = (\lambda/2\pi)\phi$, $\Delta\psi = n\lambda$ corresponds to $\Delta\phi = 2\pi n$: every image term sits at the same point on the ring, at the full circumference $2\pi R_0 = 314$ μ m for $R_0 = 50$ μ m. The $n = \pm 1$ term is therefore a winding contribution at coincidence, not a distinct half-ring feature, and there is no image at $\Delta\psi = \lambda/2$. The midpoint $\Delta\psi = \lambda/2$ is at

$$\Delta\phi = \pi, \quad \text{arc length} = \pi R_0 = 157 \text{ } \mu\text{m}, \quad (22)$$

independent of λ , where the direct $n = 0$ and $n = -1$ terms are equidistant: it is the *antipodal correlation*, not an image. Whether that correlation carries an image excess is exactly the open M0 calculation. If one insists on resolving the image term rather than the antipodal correlation, the relevant separation is the full ring $2\pi R_0$, and the thermal requirement is $T \lesssim 0.08$ nK (Eq. (24)).

For $R_0 = 50 \mu\text{m}$, neither the antipodal arc ($157 \mu\text{m}$) nor the image circumference ($314 \mu\text{m}$) is reached by the original 1–50 μm separation range.

The two channels are therefore complementary but not a joint overdetermination of the same coefficient:

- the antipodal density correlation at $\Delta\psi = \lambda/2$ tests the quotient periodicity and the prepared state;
- the mode density (and, if the damping obstruction can be circumvented, a Bragg measurement) tests the UV population and its saturation at the healing length.

The image sum controls both in principle, but the relation between the finite-separation correlation and the coincidence-limit coefficient must be derived before the two can be fitted jointly.

3.4 Design targets, not outcomes

Because the coefficient of the measured finite-separation correlation is not yet derived, and because the nominal parameters do not reach the signal (Section 6), this study does not claim a two-outcome test. It defines what would count as evidence in a future run. The targets are conditional on M0 (the image sum for the measured correlator) and on the state-preparation calculation.

Target (A): scaling with the image coefficient. The coefficient inferred from the short-distance correlation tracks the image-sum prediction S_λ , and the inferred stress follows u^{-2} (in 2D) down to $u \sim \xi$. At $u \lesssim \xi$ the Bogoliubov dispersion cuts the divergence off; the saturation scale is the healing length, and the saturation value is the one predicted by the known UV completion. This would be evidence that the kinematic image-sum mechanism operates in the analogue system, with the cutoff understood. It would be the analogue of semiclassical chronology protection working where the effective theory is valid.

Target (B): suppression or early regularization. The inferred coefficient is suppressed relative to S_λ , or the power law breaks at a scale larger than ξ . This would point to the state-choice or regularization channel: the state prepared by the collapse is not the diverging vacuum, or the condensate’s mean-field backreaction reorganizes the pile-up before the healing length is reached. It is the analogue of the Li–Gott adapted vacuum [10] and of the Barceló et al. analogue chronology protection [21]. It would be a positive result for the obstruction framework and a caution for inferences drawn from the curved cores.

A third, inconclusive category is possible: the amplitude follows S_λ at large u but the exponent drifts before ξ , indicating that the image sum is modified by the finite ring size or by nonlinearity. This would not separate (A) from (B); it would locate the scale at which the effective description breaks and would constrain the next design. The three categories should be kept distinct, and the binary “Hawking versus state-choice” language of the original draft is withdrawn.

3.5 The state-choice scan

The BEC offers a capability astrophysical tests lack: the initial state can be prepared. The default preparation is the ground state of the ring at R_0 ; the M0 calculation will determine which state on the quotient this preparation approximates. A future run could also prepare:

- a squeezed phonon state (via a parametric drive or a non-adiabatic ramp);
- a thermal state with a controlled phonon occupation;
- a phase-imprinted state with a definite angular momentum.

Each preparation corresponds to a different choice of state on the quotient. The interpretation of the scan is conditional on M0: the M0 calculation must supply the Bogoliubov transformation between the prepared state and the conformal states for which the image sum is known. If the inferred coefficient depends on the preparation in the way the transformation predicts, the state-choice channel would be under control. If it is universal, the image-sum coefficient would be robust against state choice in this geometry. Either result would sharpen the Li–Gott debate [10, 8, 9, 11, 18]; neither is a prediction of this study.

The state-choice scan would be the main scientific advantage of the BEC platform over the optical one. The optical proposal’s quantum limits are set by amplified vacuum fluctuations above a lasing threshold [20]; the BEC’s limits are set by the prepared many-body state and the healing length. The two are different windows on the same question.

4 Experimental requirements

4.1 Nominal condensate parameters

Table 3 lists a nominal parameter set for a ^{87}Rb ring. The values are design estimates, not measured values; they are chosen to keep the acoustic and one-dimensional conditions marginal at best. The scattering length $a_s = 5.3$ nm is the usual ^{87}Rb value at low field; the density and transverse confinement give $\mu < \hbar\omega_\perp$ and $T \ll T_{\text{max}}$.

The one-dimensional condition is $\mu \ll \hbar\omega_\perp$. The table gives $\mu/(\hbar\omega_\perp) = 0.45$, which is *not* $\ll 1$; a tighter transverse trap ($w = 0.12$ μm) would reduce the ratio to 0.29 at the cost of a larger transverse gap. The transverse modes are the largest single systematic and must be validated in M1 (Section 5).

The temperature requirement is now the dominant obstacle. The Bragg band at $f \gtrsim 3$ kHz would be vacuum-dominated for $T \lesssim 5$ nK. The equal-time correlation is limited by the thermal coherence length

$$\ell_T = \frac{\hbar c_s}{k_B T}, \quad (23)$$

which is 4.99 μm at 5 nK and 12.46 μm at 2 nK with the corrected $c_s = 3.263$ mm/s. The antipodal midpoint is at $\Delta\psi = \lambda/2$, an arc length $\pi R_0 = 157$ μm (Eq. (22)), independent of λ ; the $n = \pm 1$ image term is at the full ring $2\pi R_0 = 314$ μm , the same point. The criterion $\ell_T \gtrsim 2\pi R_0$ for the image term requires

$$T \lesssim \frac{\hbar c_s}{k_B 2\pi R_0} = 0.0795 \text{ nK}, \quad (24)$$

i.e. about 0.08 nK, while the weaker $\ell_T \gtrsim \pi R_0$ criterion for the antipodal correlation requires 0.159 nK, about 0.16 nK. At 2 nK the antipodal correlation is suppressed by $\exp(-\pi R_0/\ell_T) \approx 3 \times 10^{-6}$ and the image term by $\exp(-2\pi R_0/\ell_T) \approx 1 \times 10^{-11}$. The effective horizon temperature $\hbar c_s/(2\pi k_B \xi) = 25.1$ nK is the analogue Hawking/Unruh scale of the healing-length cutoff; it is not the scale that limits the correlation, and it is more than two orders of magnitude above the 0.08 nK image requirement. Cooling a 50 μm ring to 0.08 nK is well beyond demonstrated ring-BEC technology, and the in-situ separation range of 1–50 μm reaches neither the antipodal arc (157 μm) nor the image circumference (314 μm) in any case. The only geometric escape is a much smaller ring, $R_0 \simeq 5$ μm , which puts the image term at $2\pi R_0 = 31.4$ μm (the antipodal arc is 15.7 μm) and lowers the image requirement to $T \lesssim 0.8$ nK; it reduces the atom number to $N \sim 670$ and the mode count to $R_0/\xi \sim 32$, making the continuum image sum marginal.

Table 3: Nominal parameters for the design ^{87}Rb ring. All values are estimates for design purposes; a collaborating lab will optimize them. The healing length and sound speed follow from Eqs. (7)–(8).

Quantity	Symbol	Nominal value
Atom	^{87}Rb	–
Peak density	n	$3 \times 10^{20} \text{ m}^{-3}$
Scattering length	a_s	5.3 nm
Healing length	ξ	0.158 μm
Sound speed	c_s	3.263 mm s^{-1}
Chemical potential	μ/k_B	112 nK
Transverse width	w	0.15 μm
Transverse frequency	$\omega_{\perp}/2\pi$	5.2 kHz
Transverse gap	$\hbar\omega_{\perp}/k_B$	248 nK
Temperature (IR image)	T	$\lesssim 0.08$ nK
Temperature (vacuum Bragg band)	T	$\lesssim 5$ nK
Effective horizon temperature	$\hbar c_s/(2\pi k_B \xi)$	25.1 nK
Ring radius (start)	R_0	50 μm
Ring radius (stop)	$R_{\min}$	$10\xi = 1.58 \mu\text{m}$
Atom number at R_0	$N(R_0)$	$\sim 6.7 \times 10^3$
Atom number at $R_{\min}$	$N(R_{\min})$	$\sim 2.1 \times 10^2$
Rapidity range	λ	1– 2π
Sweep time	t_{sweep}	15–96 ms

4.2 The collapse protocol

The ring is produced by a time-averaged optical potential or a magnetic ring trap. The radius is controlled by the position of the inner and outer walls; a linear ramp $R(t) = R_0 - Vt$ is imposed with $V = \lambda c_s/(2\pi)$. For $\lambda = 1$, $V = 0.519 \text{ mm/s}$ and the 50 μm ring would collapse in 96 ms (93 ms to $R_{\min}$). For $\lambda = 2\pi$, $V = c_s = 3.263 \text{ mm/s}$ and the collapse takes 15.3 ms (14.8 ms to $R_{\min}$); this is the sonic limit, where the condensate can no longer adjust quasi-statically, and it is a stretch goal rather than the baseline. The baseline run uses $\lambda = 1$: the wall velocity is $0.16c_s$, and the sound-crossing time $R_0/c_s = 15.3 \text{ ms}$ is a factor 6.3 shorter than the 96 ms sweep. That is marginal, not a comfortable adiabatic separation; it must be checked by the M1 Gross–Pitaevskii simulation. The sweep is *not* short compared with the relevant phonon lifetimes for the UV modes (Section 5.3).

The sound speed must be held constant during the ramp. If the density changes as the ring shrinks, $c_s \propto \sqrt{n}$ changes and the effective metric is no longer exactly Milne. Three mitigations are available: (i) compensate the scattering length with a Feshbach resonance so that na_s is constant; (ii) keep the three-dimensional density constant by reducing the atom number in proportion to R through a reservoir; (iii) adjust the transverse width so that the volume, and hence n , stays constant. The simplest protocol is (ii): a reservoir removes atoms as the ring shrinks, keeping n and c_s fixed. The M1 Gross–Pitaevskii simulation will fix the optimal ramp and quantify the residual density variation.

The relative calibration of λ is the key drive requirement. Since $\lambda = 2\pi V/c_s$, a 1% measurement of V and c_s gives a 1% calibration of λ , not 10^{-3} . Reaching $\delta\lambda/\lambda \sim 10^{-3}$ would require c_s to 0.1%, which is not demonstrated here. The original draft’s 10^{-3} target and its “relative phase 10^{-3} ” sentence are withdrawn: the radius ramp has no optical phase, and the phase requirement belonged

Table 4: Corrected values. The left column is the original draft; the right column is the value implied by $c_s = \sqrt{gn/m} = 3.263 \text{ mm/s}$ and the geometry $\psi = (\lambda/2\pi)\phi$. Velocities, temperatures and frequencies are multiplied by $1/\sqrt{2}$ and times by $\sqrt{2}$; lengths are set by the ring radius. The original draft conflated the antipodal correlation with the image, so the last four rows separate them.

Quantity	Original draft	Corrected
Sound speed c_s	4.62 mm s ⁻¹	3.263 mm s ⁻¹
$V(\lambda = 1)$	0.735 mm s ⁻¹	0.519 mm s ⁻¹
$V(\lambda = 2\pi)$	4.62 mm s ⁻¹	3.263 mm s ⁻¹
Sweep to $R_{\min}$, $\lambda = 1$	65.9 ms	93.2 ms
Sweep total, $\lambda = 1$	68.0 ms	96.3 ms
Sweep to $R_{\min}$, $\lambda = 2\pi$	10.5 ms	14.8 ms
Sweep total, $\lambda = 2\pi$	10.8 ms	15.3 ms
$T_{\text{horizon}} = \hbar c_s / (2\pi k_B \xi)$	35.5 nK	25.1 nK
Bragg $f = c_s / (2\pi \xi)$	4.65 kHz	3.28 kHz
$\hbar f / k_B$ at that f	223 nK	158 nK
$\ell_T(5 \text{ nK})$	7.06 μm	4.99 μm
$\ell_T(2 \text{ nK})$	17.6 μm	12.46 μm
R_0/c_s (sound crossing)	10.8 ms	15.3 ms
Antipodal arc ($\Delta\psi = \lambda/2$)	25 μm	$\pi R_0 = 157 \mu\text{m}$
First-image arc ($n = \pm 1$, $\Delta\phi = 2\pi$)	25 μm	$2\pi R_0 = 314 \mu\text{m}$
T for $\ell_T \gtrsim \pi R_0$ (antipodal)	2 nK	0.16 nK
T for $\ell_T \gtrsim 2\pi R_0$ (image)	2 nK	0.08 nK

to a different (moving-lattice) design. Interferometric wall monitoring can make V precise, but the sound-speed calibration sets the floor.

4.3 Detection

Three detection channels were considered. Two survive as measurements; the third is obstructed at the nominal parameters.

(i) *In-situ density correlation.* Phase-contrast or absorption imaging with $\sim 0.5 \mu\text{m}$ resolution along the ring. The ring circumference at $R_0 = 50 \mu\text{m}$ is $314 \mu\text{m}$, so the image contains ~ 600 independent resolution elements. The correlation $G(\Delta\phi)$ must be extracted over the full half-ring, $\Delta\phi \in (0, \pi)$, i.e. separations up to the antipodal arc $\pi R_0 = 157 \mu\text{m}$ (Eq. (22)); the $n = \pm 1$ image term is at the full ring, $2\pi R_0 = 314 \mu\text{m}$, i.e. at coincidence. The original 1–50 μm separation range misses both. This is the primary IR measurement, and its feasibility is limited by the 0.08 nK image temperature requirement (Section 5.4), not by the imaging resolution.

(ii) *Time-of-flight angular distribution.* Release the condensate and image the momentum distribution after 10–20 ms of expansion. The angular momentum modes of the ring map to radial momentum peaks; the mode occupations n_m are read directly. This channel measures the mode density populated by the collapse. It does not by itself measure the image-sum coefficient, because the map from n_m to the coincidence-limit stress is an M0 calculation.

(iii) *Bragg spectroscopy.* Two counterpropagating laser beams with momentum transfer $2k_L \simeq 1.6 \times 10^7 \text{ m}^{-1}$ (for 780 nm light) probe the dynamic structure factor $S(q, \omega)$. At $q \sim 2.5/\xi$ the Bogoliubov frequency is $f \simeq 16.7 \text{ kHz}$, not the 0.5–10 kHz of the original draft, and at $q \sim 1/\xi$ it is $f \simeq 4.0 \text{ kHz}$. A fixed- q frequency scan resolves the discrete Bogoliubov peaks of the finite ring, not the log-periodic comb of Eq. (19); and the Beliaev linewidth at $k\xi \sim 2.5$ is a few kHz (Section 5.3). The Bragg comb is therefore not a usable observable in this design. A future broadband or q -

scanning scheme with a shorter pulse and a lower- k probe would be needed.

The three channels are not equally useful. (i) is the primary correlation measurement and the least UV-sensitive; (ii) measures the mode density and is the natural companion to (i); (iii) is the most UV-sensitive but is obstructed by discrete-mode sampling and Beliaev damping, and is not part of the baseline design.

4.4 Shot budget and sensitivity

The stress coefficient varies strongly over the proposed range. Over $\lambda \in [1, 2\pi]$, S_λ changes by a factor

$$\frac{S_\lambda(1)}{S_\lambda(2\pi)} = \frac{2.373}{3.756 \times 10^{-3}} \simeq 632, \quad (25)$$

and over $R_0/R_{\min} = 31.6$ the $1/u^2$ law changes the stress by a factor $\simeq 10^3$. These are variations of the *stress coefficient*, not of the finite-separation density correlation $G(\Delta\phi)$. The original draft used them to argue that a 10% measurement of the correlation amplitude would suffice; that inference is invalid, because the coefficient of G at finite separation is not S_λ and has not been computed.

What can be said is narrower. The λ -dependence of the inferred coefficient is a parameter-free prediction once M0 fixes the relation between G and T_{uu} ; the shot budget must be set against that prediction when it exists. With N_{shots} shots per setting, $1/\sqrt{N_{\text{shots}}} = 3.2\%$ for 10^3 shots only if the correlation is shot-noise-limited and the ~ 600 resolution elements are independent. Imaging response, atom-number fluctuation, and the 10–20% systematics of Table 5 dominate that number. The shot noise is not the limiting error and the original “comfortably below the predicted variation” is withdrawn.

The fit would have three parameters: an overall amplitude, the exponent α in $u^{-\alpha}$, and the saturation scale u_{sat} . The amplitude is predicted only after M0; the exponent is predicted to be 2 in 2D for the stress; the saturation scale is predicted to be ξ in target (A) and larger in target (B). A joint fit to the R and λ dependence would overdetermine the model only if the G – T_{uu} relation is known, so the overdetermination claim is also conditional on M0.

5 Systematics

Table 5 collects the estimated systematics. It is not a quantified error budget: the status column distinguishes calculations, design requirements, and placeholders that M1/M2 must measure. The entries do not establish that the systematics are under control, and they are not used as evidence of feasibility.

5.1 Finite size and mode discreteness

The image sum (14) is a continuum sum. On a ring, the angular modes are discrete and the sum is truncated at $n_{\max} \sim R/\xi$. At $R_0 = 50 \mu\text{m}$ and $\xi = 0.158 \mu\text{m}$, $R_0/\xi = 316$; at $R_{\min} = 10\xi$, only ten modes remain. If the density is held constant by removing atoms (protocol ii), $N \propto R$ and $N(R_{\min}) \approx 2.1 \times 10^2$, against $N(R_0) \approx 6.7 \times 10^3$. The continuum approximation is therefore good at the start and marginal at the stop; a 212-atom ring at $R = 1.58 \mu\text{m}$ may not support the required phonon modes at all. The fit must use the discrete mode sum, not the continuum integral, and the last few points should be treated as a finite-size-limited measurement of the saturation, not of the asymptotic exponent. Ring stability at $R_{\min}$ is an M1 deliverable, not a result.

5.2 Transverse modes

The one-dimensional reduction requires the transverse gap $\hbar\omega_\perp$ to exceed the chemical potential and the temperature. The nominal ratio is 0.45, which gives a 20% estimate for the leading transverse correction. The correction is not a random error: it is a calculable shift of the effective coupling and the sound speed. The M1 simulation will compute it for the actual trap; a future run could vary w and extrapolate to the 1D limit. A tighter trap ($w = 0.12 \mu\text{m}$) reduces the ratio to 0.29 but raises the transverse gap to 388 nK, which is acceptable.

5.3 Phonon lifetime and Beliaev damping

The phonon lifetime is mode-dependent, and the original draft's global estimate $\tau_{\text{ph}} \sim 0.1\text{--}1 \text{ s}$ was wrong for the modes the Bragg channel probes. At $T = 0$ a homogeneous BEC damps phonons by Beliaev decay (a phonon splitting into two), which is not suppressed by the thermal fraction. For $k\xi \ll 1$ the standard rate is

$$\Gamma_k \sim \frac{3}{640\pi} (k\xi)^5 \omega_k. \quad (26)$$

At $k\xi \sim 1$ this gives $\Gamma \sim 40 \text{ s}^{-1}$, $\tau \sim 25 \text{ ms}$; at $k\xi \sim 2.5$, extrapolating the same estimate, $\Gamma \sim 1.5 \times 10^4 \text{ s}^{-1}$, $\tau \sim 50\text{--}70 \mu\text{s}$. Both are shorter than or comparable to the 96 ms baseline sweep, and the $k\xi \sim 2.5$ mode is far shorter than the 0.1–1 ms Bragg pulse and has a linewidth $\Gamma/2\pi$ of a few kHz. The low- k modes that dominate the IR correlation may be much longer-lived, but that must be shown mode by mode; a global τ_{ph} cannot be quoted.

Three-body loss is a separate process and is not phonon damping. With $K_3 \approx 5.8 \times 10^{-42} \text{ m}^6 \text{ s}^{-1}$ and $n = 3 \times 10^{20} \text{ m}^{-3}$, $K_3 n^2 \approx 0.5 \text{ s}^{-1}$, not 0.1 s^{-1} ; and atom loss removes particles without necessarily destroying phase coherence. The lifetime requirement $\tau_{\text{ph}} > 3t_{\text{sweep}}$ of the original draft is therefore not met for the UV/Bragg modes. It may be met for the low- k IR modes, but that is an open M1 measurement, not a design result.

5.4 Temperature

The vacuum-dominated character of the high-frequency band is not in doubt: at $f = c_s/(2\pi\xi) = 3.28 \text{ kHz}$, $\hbar f/k_B = 158 \text{ nK}$, so at $T = 5 \text{ nK}$ the occupation is $\exp(-158/5) \sim 10^{-14}$. What fails is the IR correlation. The equal-time correlation is limited by the thermal coherence length (23): $\ell_T \approx 4.99 \mu\text{m}$ at 5 nK and $\approx 12.5 \mu\text{m}$ at 2 nK. The antipodal midpoint is at $\Delta\psi = \lambda/2$, an arc $\pi R_0 = 157 \mu\text{m}$ (Eq. (22)), so measuring the antipodal correlation requires $\ell_T \gtrsim \pi R_0$, i.e. $T \lesssim 0.16 \text{ nK}$. The $n = \pm 1$ image term sits at the full ring $2\pi R_0 = 314 \mu\text{m}$ (the same point), so resolving it requires $\ell_T \gtrsim 2\pi R_0$, i.e. $T \lesssim 0.08 \text{ nK}$ (Eq. (24)). At 2 nK the antipodal correlation is suppressed by $\exp(-\pi R_0/\ell_T) \approx 3 \times 10^{-6}$ and the image term by $\exp(-2\pi R_0/\ell_T) \approx 1 \times 10^{-11}$.

This is a genuine blocker, not a matter of better cooling. The reference run at the same T and R but with a non-collapsing ring would subtract the static correlation background, the imaging response, and the thermal occupation, but it cannot restore a term that is exponentially suppressed. The only routes are a much smaller ring ($R_0 \simeq 5 \mu\text{m}$, which puts the image term at $2\pi R_0 = 31.4 \mu\text{m}$ and the antipodal arc at $15.7 \mu\text{m}$ but reduces N to ~ 670 and R_0/ξ to ~ 32) or a different observable at shorter separation.

5.5 Nonlinearity and the healing length

The Bogoliubov dispersion is

$$\omega^2 = c_s^2 k^2 \left(1 + \frac{1}{2} k^2 \xi^2\right), \quad (27)$$

so the relativistic analogy fails at $k\xi \sim 1$. The healing length is therefore both the UV cutoff and the scale at which the analogy is expected to break. In target (A), the break is the saturation; in target (B), the break happens earlier for a different reason. A future run should not try to subtract the healing-length cutoff; it should measure it. The cutoff scale is a prediction of the microphysics, and its location is a check on the interpretation.

The mean-field nonlinearity also provides an analogue of backreaction: the phonon stress modifies the condensate density, which modifies the effective metric. This is not Einstein gravity, but it is a self-consistent response. If the response is strong enough to smooth the pile-up before ξ , a future run would see target (B) for a reason that is specific to the BEC. The M1/M2 Gross–Pitaevskii simulation is needed to separate this from the state-choice channel.

5.6 Drive calibration and background flow

The rapidity is $\lambda = 2\pi V/c_s$. The wall velocity V can be monitored interferometrically, but the calibration is limited by the sound-speed measurement. A 1% measurement of c_s gives a 1% uncertainty in λ ; reaching $\delta\lambda/\lambda \sim 10^{-3}$ would require c_s to 0.1%, which is not demonstrated. The design target is therefore stated as a requirement, not as achieved performance. A residual azimuthal flow v_ϕ adds a Doppler shift and changes the effective identification; the requirement $v_\phi < 0.01c_s$ is met by a non-rotating ring and checked by the angular-mode spectrum.

5.7 Interpretation systematics

The most important systematic is interpretive, not instrumental. The BEC reproduces the kinematics of a 2D conformal scalar on the Milne quotient cylinder; it does not reproduce the 4D Einstein equations, the gravitational backreaction, the null horizon, or the CTC region. A measured image structure would be evidence that the kinematic pile-up mechanism is realized in the analogue system. It is not evidence that chronology protection holds or fails in quantum gravity. The paper’s claims must be stated at this level, and the comparison to the curved cores must be made through the shared image-sum machinery, not through a literal identification of the BEC with spacetime.

6 Feasibility verdict

The design as specified is *not* feasible. The failures listed below are order-of-magnitude mismatches, not refinements of an error budget. Table 6 states them explicitly.

Two routes could change the verdict, and neither is closed here. (i) *Smaller ring*. At $R_0 \simeq 5 \mu\text{m}$ the image term is at the full ring $2\pi R_0 = 31.4 \mu\text{m}$ (the antipodal arc is $15.7 \mu\text{m}$), and the image temperature requirement falls to $T \lesssim 0.8 \text{ nK}$, which is demanding. The cost is $N \approx 670$ and $R_0/\xi \approx 32$: the continuum image sum becomes marginal and the discrete-ring correction is no longer a 10% effect. Demonstrating a stable 670-atom ring at 0.8 nK is an M1 deliverable. (ii) *Lower- k or time-domain UV probe*. At $k\xi \lesssim 1$ the Beliaev lifetime is tens of ms, and a quench or Ramsey-type measurement with a short pulse could in principle resolve the mode population before it damps. Whether such a probe measures the image sum rather than the Bogoliubov dispersion is an M0 question. Until one of these routes is worked out, the honest status is a design study with open blockers, not a ready proposal.

The value of the study is that the blockers are now quantitative and separable: B1, B2, B7, B8 and B9 are fixed; B3, B4, B5 and B6 are open. A future paper should not claim feasibility until the open blockers are closed.

7 Relation to other work

7.1 The optical analogue

Bermudez and Leonhardt propose an optical experiment in which time travel is inherently related to lasing, and the amplified vacuum fluctuations above laser threshold establish the quantum limits [20]. The proposal is the closest experimental comparator: it attacks the same conjecture with a moving-medium geometry and a different amplifier. The BEC design is complementary in three respects:

1. *UV completion.* The BEC has a known cutoff, the healing length, and a controlled Bogoliubov dispersion. The optical medium has a dispersion of its own, but the lasing threshold and the gain saturation are the dominant quantum limits. The two experiments test different cutoffs.
2. *State preparation.* The BEC state is a many-body ground state that can be squeezed, heated, or phase-imprinted. The optical state is set by the laser. The BEC could therefore perform the state-choice scan of Section 3.5 (conditional on M0); the optical proposal is optimized for the amplification threshold.
3. *Observable.* The BEC measures density correlations and the angular-mode density; the optical proposal measures the lasing threshold and the amplified vacuum. A positive result in either would motivate the other, and a null result in one would constrain the interpretation of the other.

The two should be read as a joint program: the optical experiment is the faster path to a horizon-scale amplifier; the BEC design, if its blockers are closed, would be the cleaner path to the image-sum coefficient and the state-choice question.

7.2 The Ori and Xavier cores

The Ori 2005 core is a curved, asymptotically flat vacuum solution with a compact toroidal CTC region and a chronology horizon that is probably, but not proven, non-compactly generated [13]. Geometric analyses of the Ori family and the isocausality classification of Misner-type spacetimes provide the causal-structure framework [16, 17]. The companion image-series work (in preparation) reports that the stress on the core follows the Misner ϵ^{-2} law but is capped near the horizon locus, while the Xavier 2026 core has a degenerate horizon family with a structurally different winding behaviour [19, 32, 33]. We cite these only qualitatively: the numerical values are unpublished, proxy-dependent, and in the Xavier case contested. No result in this design study depends on them. The scalar-quasiresonance formulation of chronology protection [31] is a complementary instability-side diagnostic.

The relation to the BEC design is through the shared image-sum machinery, not through a literal identification of the BEC with either core. If a future experiment inferred a coefficient following S_λ and the u^{-2} law, that would be evidence that the image-sum mechanism is realized in the cleanest quotient geometry; it would not by itself settle the interpretation of the capped Ori stress or the Xavier family, because those are different (curved, 4D, and in the Xavier case degenerate) settings. Conversely, a measured suppression or early saturation would show that the state-choice or regularization channel can operate in a controlled analogue; it would not prove that the same channel operates in the curved cores. The original draft’s “laboratory calibration” language is withdrawn for this reason.

7.3 Other platforms

Superconducting circuits can simulate non-causal null curves and related kinematics [23]; they offer fast control and strong nonlinearities, but their effective field is not a simple conformal scalar and their UV completion is less clean than the BEC healing length. Optical fibers and moving media [25, 20] offer the fastest horizon dynamics but a less controllable initial state. The BEC ring would be a good compromise for the image-sum coefficient—slow enough to calibrate, clean enough to have a known cutoff, and flexible enough to prepare the state—if the temperature and lifetime blockers of Section 6 can be overcome. On the present numbers it is not.

8 Milestones

Table 7 lists the milestones. The timeline assumes a single cold-atom lab with ring-trap and Bragg infrastructure. The theory milestone M0 is a laptop calculation; M1 is a bench/drive test in a non-degenerate gas (not a BEC phonon-lifetime measurement); M2 is the condensate sweep; M3 is the analysis and the second paper. The phonon-lifetime measurement belongs to M2, where a condensate exists.

The critical path is no longer M1 but the combination of B3 (temperature) and B4 (lifetime). Even a perfect ring at $R_{\min}$ does not produce a measurable image term at $T \gtrsim 0.1$ nK, and the UV modes do not survive a 96 ms sweep. M0 is the other critical path: if the finite-separation G coefficient is not related to S_λ in a simple way, the primary observable loses its target and the design must be re-scoped. These are blockers, not risks to be absorbed by a schedule.

9 Conclusion

Misner space is the cleanest chronology-horizon geometry: locally flat, compactly generated, and exactly solvable by an image sum. A quasi-one-dimensional BEC in a collapsing ring reproduces the Milne quotient cylinder natively: the azimuthal phonons see the Milne metric, the ring is the quotient circle of the boost orbit, and the collapse approaches the shrinking quotient circle. The ring’s $R = 0$ limit, however, is a conical tip, not a null horizon; the construction does not realize the compactly generated horizon and does not test KRW in any dimension. The target observable is the equal-time density correlation $G(\Delta\phi)$; the stress coefficient $S_\lambda = 2 \sum q^n / (1 - q^n)^2$ is inferred from its short-distance behaviour, not measured at finite separation.

The corrected parameters are $c_s = 3.263$ mm/s, $\xi = 0.158$ μm , $V = 0.519$ mm/s at $\lambda = 1$, a sweep of 96 ms to $R = 0$ (93 ms to $R_{\min}$), $V = 3.263$ mm/s and a 15.3 ms sweep at $\lambda = 2\pi$, a horizon temperature 25.1 nK, and a Bragg scale $f = 3.28$ kHz (Table 4). The antipodal correlation is at $\Delta\psi = \lambda/2$ (an arc $\pi R_0 = 157$ μm) and requires $T \lesssim 0.16$ nK; the first image is the $n = \pm 1$ winding term at $\Delta\psi = \lambda$, i.e. at the full ring $2\pi R_0 = 314$ μm (the same point), and requires $T \lesssim 0.08$ nK, not 2 nK.

On these numbers the design is *not* feasible. The temperature requirement (0.08 nK for the image term, 0.16 nK for the antipodal correlation) is far below the $\sim\text{nK}$ temperatures of current ring-BEC experiments, the in-situ separation range reaches neither the antipodal arc nor the image circumference, the UV modes have a Beliaev lifetime of tens of microseconds to tens of milliseconds against a 96 ms sweep, the final ring has only $N(R_{\min}) \approx 211$ atoms and ten azimuthal modes, and a fixed- q Bragg scan cannot resolve the proposed log-periodic comb. These are stated explicitly, with the corrections and scope changes, as items B1–B9 in Table 6. The paper is therefore a design study and a feasibility analysis, not a ready proposal, and it claims no measurement.

The scientific question remains worth pursuing: whether the kinematic image-sum mechanism that chronology protection relies on is realized in the simplest quotient geometry, and whether a physically prepared state follows the diverging vacuum or an adapted one. To answer it, the smaller-ring route ($R_0 \sim 5 \mu\text{m}$, $T \lesssim 0.8 \text{ nK}$ for the image, $N \sim 670$) or a lower- k time-domain UV probe must first be shown to work, and M0 must supply the finite-separation image sum. Until then, no feasibility claim is warranted.

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References

- [1] C. W. Misner, “Taub-NUT space as a counterexample to almost anything,” in *Relativity Theory and Astrophysics I: Relativity and Cosmology*, Lectures in Applied Mathematics, Vol. 8, edited by J. Ehlers (American Mathematical Society, Providence, RI, 1967), p. 160–169.
- [2] S. W. Hawking, “Chronology protection conjecture,” *Phys. Rev. D* **46**, 603 (1992).
- [3] B. S. Kay, M. J. Radzikowski, and R. M. Wald, “Quantum field theory on space-times with a compactly generated Cauchy horizon,” *Commun. Math. Phys.* **183**, 533 (1997); arXiv:gr-qc/9603012.
- [4] W. A. Hiscock and D. A. Konkowski, “Quantum vacuum energy in Taub-NUT (Newman-Unti-Tamburino)-type cosmologies,” *Phys. Rev. D* **26**, 1225 (1982). doi:10.1103/PhysRevD.26.1225.
- [5] S.-W. Kim and K. S. Thorne, “Do vacuum fluctuations prevent the creation of closed timelike curves?” *Phys. Rev. D* **43**, 3929 (1991).
- [6] G. Klinkhammer, “Vacuum polarization of scalar and spinor fields near closed null geodesics,” *Phys. Rev. D* **46**, 3388 (1992).
- [7] T. Tanaka and W. A. Hiscock, “Chronology protection and quantized fields: nonconformal and massive scalar fields in Misner space,” *Phys. Rev. D* **49**, 5240 (1994).
- [8] S. V. Krasnikov, “Quantum stability of the time machine,” *Phys. Rev. D* **54**, 7322 (1996); arXiv:gr-qc/9508038. doi:10.1103/PhysRevD.54.7322.
- [9] M. J. Cassidy, “Divergences in the effective action for acausal spacetimes,” *Class. Quantum Grav.* **14**, 3031 (1997); arXiv:gr-qc/9705075. doi:10.1088/0264-9381/14/11/006.
- [10] L.-X. Li and J. R. Gott, “A self-consistent vacuum for Misner space and the chronology protection conjecture,” *Phys. Rev. Lett.* **80**, 2980 (1998); arXiv:gr-qc/9711074. doi:10.1103/PhysRevLett.80.2980.
- [11] C. R. Cramer and B. S. Kay, “The thermal and two-particle stress-energy must be ill-defined on the 2-d Misner space chronology horizon,” *Phys. Rev. D* **57**, 1052 (1998); arXiv:gr-qc/9708028.

- [12] M. Visser, “The quantum physics of chronology protection,” in *The Future of Theoretical Physics and Cosmology* (Cambridge University Press, Cambridge, 2003); arXiv:gr-qc/0204022.
- [13] A. Ori, “A class of time-machine solutions with a compact vacuum core,” *Phys. Rev. Lett.* **95**, 021101 (2005); arXiv:gr-qc/0503077.
- [14] R. Emparan and M. Tomašević, “Holography of time machines,” *J. High Energy Phys.* **03**, 212 (2022); arXiv:2107.14200. doi:10.1007/JHEP03(2022)212.
- [15] R. Emparan and M. Tomašević, “Quantum backreaction on chronology horizons,” *J. High Energy Phys.* **02**, 182 (2022); arXiv:2109.03611. doi:10.1007/JHEP02(2022)182.
- [16] J. Dietz, A. Dirmeier, and M. Scherfner, “Geometric analysis of Ori-type spacetimes,” arXiv:1201.1929 (2012).
- [17] N. E. Rieger, “Causal classification of pathological Misner-type spacetimes,” *Annals of Physics* **489**, 170455 (2026); arXiv:2501.00841. doi:10.1016/j.aop.2026.170455.
- [18] W. Melton, A. Strominger, and T. Wang, “Quantum fields on time-periodic $\text{AdS}_3/\mathbb{Z}$,” arXiv:2510.15036 (2025).
- [19] S. Xavier, “Closed timelike curves from a vacuum traveling wave,” arXiv:2607.00788 (2026).
- [20] D. Bermudez and U. Leonhardt, “Optical time travel: proposal for testing Hawking’s Chronology Protection Conjecture in an optical analogue,” *Class. Quantum Grav.* **43**, 14LT01 (2026); arXiv:2607.22056.
- [21] C. Barceló, J. Eguia Sánchez, G. García-Moreno, and G. Jannes, “Chronology protection implementation in analogue gravity,” *Eur. Phys. J. C* **82**, 299 (2022); arXiv:2201.11072. doi:10.1140/epjc/s10052-022-10275-3.
- [22] C. Barceló, S. Liberati, and M. Visser, “Analogue gravity,” *Living Rev. Relativ.* **29**, 2 (2026). doi:10.1007/s41114-026-00064-9.
- [23] C. Sabín, “Analogue non-causal null curves and chronology protection in a dc-SQUID array,” arXiv:2207.14164 (2022).
- [24] J. Steinhauer, “Observation of quantum entanglement of Hawking radiation and its partners,” *Nat. Phys.* **15**, 961 (2019); arXiv:1809.00932.
- [25] T. G. Philbin, C. Kuklewicz, S. Robertson, S. Hill, F. König, and U. Leonhardt, “Fiber-optical analog of the event horizon,” *Science* **319**, 1367 (2008); arXiv:0707.3235.
- [26] P. Gao, D. L. Jafferis, and A. Wall, “Traversable wormholes via a double trace deformation,” *J. High Energy Phys.* **12** (2017) 151; arXiv:1608.05687.
- [27] J. Maldacena and A. Milekhin, “Humanly traversable wormholes,” *Phys. Rev. D* **103**, 066007 (2021); arXiv:2008.06618.
- [28] J. B. Hartle, “Quantum mechanics in nonchronal spacetimes,” *Phys. Rev. D* **49**, 6543 (1994); arXiv:gr-qc/9309012.
- [29] L. H. Ford and T. A. Roman, “Averaged energy conditions and quantum inequalities,” *Phys. Rev. D* **51**, 4277 (1995); arXiv:gr-qc/9412030.

- [30] C. J. Fewster, “Lectures on quantum energy inequalities,” arXiv:1208.5399 (2012).
- [31] D. Senjaya, “Probing the chronology protection conjecture via scalar quasinresonance,” *Eur. Phys. J. C* **86**, 34 (2026). doi:10.1140/epjc/s10052-026-15299-7.
- [32] J. Schemoul, “Protection without teeth: semiclassical chronology protection on the Ori time machine,” in preparation (2026).
- [33] J. Schemoul, “Quotient structure and on-generator diagnostics of vacuum traveling-wave time machines,” in preparation (2026).

Table 5: Systematic budget for the design. “Nominal size” is the estimated fractional bias on the correlation amplitude or the exponent; “status” distinguishes a calculation, a requirement, and a placeholder to be measured in M1/M2. This is a list of estimates and requirements, not a demonstrated budget, and it is not evidence that the systematics are under control.

Effect	Nominal size	Status	Mitigation / requirement
Finite ring / mode discreteness	$\xi/R_{\min} \sim 10\%$ at $R_{\min} = 10\xi$	calculation; $N(R_{\min}) \sim 210$, ten modes	fit only $R \geq 10\xi$; use the discrete mode sum; extend R_0
Transverse modes	$\mu/(\hbar\omega_{\perp}) = 0.45$; bias $\sim 20\%$	estimate	tighter trap ($w = 0.12 \mu\text{m}$) or lower density; measure the transverse gap
Phonon lifetime (intermediate modes)	$\tau \sim 25 \text{ ms}$ at $k\xi \sim 1$	Beliaev calculation	shorter than the 96 ms sweep; mode-resolve
Phonon lifetime (Bragg modes)	$\tau \sim 50 \mu\text{s}$ at $k\xi \sim 2.5$; $\Gamma/2\pi \sim \text{few kHz}$	Beliaev calculation	<i>fails</i> the pulse and sweep requirement; Bragg comb withdrawn
Temperature	$T \lesssim 0.08 \text{ nK}$ (IR image); 0.16 nK (antipodal); $\lesssim 5 \text{ nK}$ (Bragg)	requirement	not demonstrated for a $50 \mu\text{m}$ ring; smaller ring trade
Nonlinearity / healing length	cutoff at $u \sim \xi$; Bogoliubov dispersion	calculation	include $k^2\xi^2$; treat the cutoff as a feature
Drive calibration	$\delta\lambda/\lambda \sim 1\%$	requirement	interferometric wall monitoring; c_s to 0.1% for 10^{-3}
Atom loss	$K_3 n^2 \sim 0.5 \text{ s}^{-1}$ for $K_3 \approx 5.8 \times 10^{-42}$	estimate; loss is not phonon damping	low density; short sweep; correct for measured loss
Imaging resolution	$0.5 \mu\text{m} \approx 3\xi$	calculation	in-situ over the half-ring; no UV imaging
Background flow	$v_{\phi} < 0.01c_s$	requirement	no deliberate circulation; measure the Doppler shift
Shot noise	$1/\sqrt{N_{\text{shots}}} \sim 3\%$ for 10^3 shots	arithmetic only; systematics dominate	10^3 – 10^4 shots per setting
Analog interpretation	kinematic only	scope statement	no “CTC created” language; compare only to the effective-field prediction

Table 6: Blockers for the nominal design. Each is a quantitative mismatch, not an unmodelled systematic.

Item	Blocker	Consequence
B1	The original draft used $c_s = \hbar/(m\xi) = 4.62$ mm/s; the correct value is 3.263 mm/s.	The velocities, temperatures and frequencies shift by $1/\sqrt{2}$ and the times by $\sqrt{2}$; corrected in Table 4. Fixed.
B2	S_λ is the coincidence-limit stress coefficient, not the finite-separation density-correlation amplitude.	The measured G and the inferred S_λ are now separated; the G coefficient is an M0 calculation. Fixed as a definition; the prediction is open.
B3	The $n = \pm 1$ image is at $\Delta\psi = \lambda$, i.e. $\Delta\phi = 2\pi$ and the full ring $2\pi R_0 = 314 \mu\text{m}$ (the same point), requiring $T \lesssim 0.08$ nK for its thermal coherence; the antipodal correlation at $\Delta\psi = \lambda/2$ ($\pi R_0 = 157 \mu\text{m}$) requires $T \lesssim 0.16$ nK and is not itself an image.	The 50 μm ring is not feasible. A 5 μm ring lowers the image requirement to $T \lesssim 0.8$ nK but leaves $N \sim 670$ and $R_0/\xi \sim 32$. Open.
B4	Beliaev damping gives $\tau \sim 50 \mu\text{s}$ at $k\xi \sim 2.5$ and ~ 25 ms at $k\xi \sim 1$, shorter than the 96 ms sweep.	The UV modes do not survive the sweep; the global 0.1–1 s lifetime is withdrawn. Open.
B5	A fixed- q Bragg scan resolves discrete Bogoliubov peaks, not a log-periodic frequency comb; combined with B4 the comb is washed out.	The spectral-comb observable is withdrawn. Open; needs a different scheme.
B6	At $R_{\min} = 10\xi$ the ring has only $N(R_{\min}) \approx 2.1 \times 10^2$ atoms and ten azimuthal modes, against $N(R_0) \approx 6.7 \times 10^3$ and $R_0/\xi = 316$ at the start.	The continuum image sum is marginal at the stop; a 212-atom ring may not support the required phonon modes. Ring stability is an M1 deliverable. Open.
B7	$R = 0$ is a conical tip, not a compactly generated null horizon; the CTC side and the closed null geodesics are absent.	The construction is a kinematic analogue of the quotient cylinder and does not test KRW in any dimension. Fixed as a scope statement.
B8	The original title, abstract and conclusion claimed a laboratory test, a first measurement, and a calibration of the curved cores.	Claims are now “design study”, “kinematic analogue”, and “target”. Fixed.
B9	The Ori/Xavier relation used unpublished, proxy-dependent companion numbers.	Those numbers are removed; the relation is qualitative and model-dependent. Fixed.

Table 7: Milestones and decision points. Durations are estimates.

Item	Deliverable	Duration	Decision
M0	Phonon image sums on the Misner quotient: exact 2D coefficient, normalization against the $\lambda = 2\pi$ Rindler anchor, 3D extension, and—critically—the derivation of $G(\Delta\phi)$ from the image sum and its relation to S_λ .	4 weeks (laptop)	Is the finite-separation G coefficient related to S_λ , and does the discrete ring sum survive?
M1	Collapsing-ring drive in a non-degenerate gas: linear radius ramp, wall-velocity calibration, Gross–Pitaevskii simulation of the density response, and demonstration of ring stability at small R .	2–3 months	Is the ring stable at $R_{\min} = 10\xi$ with $N \sim 210$?
M2	Condensate sweep: $\lambda \in [1, 2\pi]$, density correlation over the half-ring, angular-mode occupations, and a mode-resolved measurement of τ_{ph} (Beliaev).	6–9 months	Is the inferred coefficient consistent with S_λ or suppressed?
M3	Analysis and second paper: fit of amplitude, exponent, and saturation scale, conditional on M0; state-choice scan.	3 months	Which target, (A), (B), or inconclusive?