

Initial data for a time machine from dust: an explicit composite construction with a certified ordinary-density scaling regime

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Abstract

We construct explicitly, and verify numerically to a documented fine-grid standard, the complete initial data of an asymptotically flat composite in the Ori program: a vacuum pp-wave core with boost-identified z -circles, wrapped in a dust envelope. The result is initial data for a time machine, not a manufactured time machine: its evolution into closed timelike curves is conditional on Ori's existence theorem, which we cite rather than re-prove. The construction is explicit layer by layer; each quantitative statement is closed-form (the profiles), interval-certified (the post-relaxation Ricci bound), exhibited (fine-grid gates on the gatefast interior band), explicitly conjectured (the full open-band $O(a)$ uniformity), extrapolated (the canonical family's deep-rung mass and density), or cited: an umbilical slice family with pure-trace extrinsic curvature $K = k_0 h$ (exact at $a = 0$); an envelope layer joining it to Ori's dust layers under both gates ($H_1 \geq 0$ and $H_1 \geq 2|M|$), robust under $\pm 30\%$ perturbations of every profile parameter on the gatefast interior band $r \geq w + 0.0904$ of a fixed $\Delta r = 0.005$, 48ϕ grid, with sampled margins $+0.763129$ and $+0.728189$ at $a = 0.02$ and interval-certified lower bounds $H_1 \geq 0.7468$, $H_1 - 2|M| \geq 0.2736$ on that band; the retuned construction has both gates positive over the full envelope layer $r > w$, including the innermost collar, at $a = 0.02$ (the exact vacuum boundary $r = w$ is marginal, with $WEC(1\text{ mm}) \geq +4.55 \times 10^{-6}$ and a dense exact-jet scan positive on $r > w$); a two-stage flattening onto Ori's Layer-2 form (verified: $H_1 = 6k_0^2$ to five decimals); and Ori's outer layers, re-implemented and re-verified here. The composite is asymptotically flat through Ori's exterior layer; we rely on that construction and do not independently compute its ADM charges. The dust

of the title is literal, but the canonical corner is not ordinary matter: $\varepsilon_0 = 4.0 \times 10^{25} \text{ kg/m}^3$ (peak $1.17 \times 10^{26} \text{ kg/m}^3$, $\sim 5 \times 10^8$ times nuclear saturation). The dust becomes ordinary-density only at the deep rung $|k_0| \simeq 2.2 \times 10^{-12}$ ($\varepsilon_0 = 778 \text{ kg/m}^3$). For the canonical family that rung is reached by extrapolating the certified k_0^2 density and $|k_0|$ -linear mass laws; a separately retuned deep family is directly gate-evaluated there by `massdial_verify.py`. At fixed ρ_0 and dimensionless z -period L_z the C4-layer mass scales as $|k_0|$ (not k_0^2): $M_{\text{layer}} = 3.30 |k_0| L_z$ in geometrized units ($9.8 \times 10^{15} \text{ kg } L_z$ at the deep rung); adding the RW tube gives $1.3\text{--}2.4 \times 10^{16} \text{ kg } L_z$, but the linear law is demonstrated only for the C4 layer and the RW tube, not for the full upper envelope that includes Ori's outer layer. The scaling group is $(\rho_0, \tau_0, \ell_i) \rightarrow (\lambda \rho_0, \lambda^2 \tau_0, \lambda \ell_i)$ at fixed (a, e, L_z) , confirmed to 10^{-8} . The open-band $O(a)$ uniformity of the fixed-knob family is stated as an explicit conjecture; an interval-certified derivative enclosure covers $a \leq a_W^* = 0.0346$ (still > 0.02) on the interior band.

1 Introduction

Ori's time-machine program [1, 2] combines a vacuum core with closed boost-identified z -circles (the 2005 plane-wave solution [1]) with a dust envelope [2] completing it to an asymptotically flat initial-data set (2007). The 2007 construction builds its envelope around pseudo-Schwarzschild boundary data; transplanting it to the 2005 core requires (i) a hypersurface in the core on which K_{ab} is pure trace, (ii) an explicit envelope layer realizing the constraint inequalities against the core's boundary data, and (iii) the matching to the 2007 outer layers. This paper delivers all three, layer by layer, with every quantitative claim tied to a status in the ledger of Sec. 7: closed form (the explicit profiles), certified (interval arithmetic), exhibited (fine-grid, machinery validated to 10^{-16}), explicitly conjectured, extrapolated (the canonical family's deep-rung mass), or cited. Asymptotic flatness is inherited from Ori's exterior layer; we do not independently recompute its ADM charges. The layer *gates* are numerical; the profiles are explicit (Appendix A).

2 Conventions and the validation standard

With dust source ε and 4-velocity u^a , the constraint data (h_{ab}, K_{ab}) on a slice satisfy

$$H_1 \equiv R^{(3)} + K^2 - K_{ij} K^{ij} = 16\pi\varepsilon, \quad M^i \equiv D_j(K^{ij} - h^{ij}K) = 8\pi\varepsilon u^i. \quad (1)$$

The Hamiltonian constraint is $H_1 = 16\pi\varepsilon$ and the momentum constraint is $M^i = 8\pi\varepsilon u^i$ (equivalently $M^i = 0$ for the normal-observer dust slicing used here). The physical requirements are the two gates $H_1 \geq 0$ (non-negative density) and $H_1 \geq 2|M|$ (the dust 4-velocity is subluminal, $|u_i| \leq 1$); we call them the Hamiltonian gate and the subluminality (WEC) gate. For $K = k_0 h$ with constant k_0 : $M^i \equiv 0$ and $H_1 = R^{(3)} + 6k_0^2$.

Validation standard. Every C4 gate claim on the envelope interior is evaluated on the gatefast interior band $r \geq w + 0.0904$ (the actual **gatefast** grid) with a fixed grid $\Delta r = 0.005$ and 48 azimuths; the innermost collar uses the exact analytic jets (no finite differences), and the flattening bands use their own fine grids. The machinery (einsum-vectorized Ricci and momentum) agrees with an independent scalar implementation to 10^{-16} at random points. Where the profiles are closed-form we go further and *certify*: the warped-product Ricci

$$R^{(3)} = -\frac{2[F^2 L_{,rr} + F L F_{,rr} + F F_{,r} L_{,r} + L_{,\phi\phi}]}{F^2 L} \quad (2)$$

(for $h = \text{diag}(1, F(r)^2, L(r, \phi)^2)$; it reduces to Ori's $-2\Delta L/L$ in the flat-base limit) is evaluated in interval arithmetic over subdivided cells. The standard exists because three earlier gate claims in this program died on too-coarse grids; the history is documented in the repository.

3 The core and the umbilical slice

The core is the Ori 2005 vacuum pp-wave with parameters (a, e) and identification period L_z in z . Its slices admit the one-parameter family

$$\tau(x, y) = \tau_0 + (e - \tfrac{1}{4})(x^2 + y^2) - \tfrac{a}{2}(x^2 - y^2), \quad 4k_0^2\tau_0 = -1, \quad (3)$$

on which $K_{ab} = k_0 h_{ab}$ *exactly* at $a = 0$ (the slice is a Minkowski hyperboloid: umbilical to 10^{-10} , k_0 exact to ten decimals) with residual shear — the largest deviation of $h^{-1}K$ from pure trace — $0.5247a$ at $\tau_0 = -1$, i.e. the quoted $1.08a/(2|\tau_0|)$ to 3%: linear in a over five decades and independent of e . At fixed ρ_0 the depth trend is closer to $a/\sqrt{|\tau_0|}$ than to $a/|\tau_0|$ (an eigenvalue audit), so depth improves the residual more slowly than the $1/|\tau_0|$ law suggests. Both dials ($a \rightarrow 0$, $|\tau_0| \rightarrow \infty$) are free core parameters: the pure-trace boundary condition C3 of the transplant program is solved in closed form. The exact data is vacuum, so the boundary torus T_0 is *marginal* ($\varepsilon = 0$, $u = n$) — the same boundary Ori's own construction faces;

the envelope must lift $\varepsilon > 0$ inward. Its two $O(a)$ inputs are measured: the shape residual above, and the truncation $\Delta(R^{(3)} + 6k_0^2) = \pm 1.25 a \rho^2 / \tau_0^2$.

4 The envelope construction

On a z -invariant chart (r, ϕ, z) with r the outward distance from T_0 (transverse radius $\rho = \rho_0 + r$), the data is

$$h = \begin{pmatrix} 1 & 0 & C(r) \\ 0 & F(r)^2 & 0 \\ C(r) & 0 & L(r, \phi)^2 \end{pmatrix}, \quad (4)$$

$$K = \text{blend of the exact core } K \text{ and } k_0 h. \quad (5)$$

Collar $r \leq w$: the exact vacuum continuation — both constraints vanish identically ($H_1 = 0$, $M^i = 0$ to machine precision): the matching statement itself, no thin shell. *Departure*: three C^2 -blended levers — the tilt decay $C = (\rho/2)(1 - S_7)$ with a quartic-onset blend S_7 (the only lever with a profile mismatch at w ; the quartic onset makes its $R^{(3)}$ response engage as $(r - w)^3$); the concave-bounded F -lever, $F_t = (\rho_0 + w) + r_F - f_c r_F^2 / 2(L_0 + r_F / \lambda_m)$, whose $F'' < 0$ lifts $R^{(3)}$ via $-2F''/F$ while the slope stays positive (no circle collapse); and an L -target derivative-matched at w . *K-relaxation in two stages*: first the tracefree part of the core K (to $k_c(r, \phi)h$), then the trace ($k_c \rightarrow k_0$) — both bands early and wide, where the established dust density dominates the momentum budget $|M| \sim 1.2 |\Delta K|/\text{width}$.

Design point and knobs. The canonical design in the program notes is

$$(\rho_0, e, a, \tau_0, L_z) = (0.5 \text{ m}, 0.1, 10^{-4}, -1 \text{ m}^2, 1);$$

all exhibited gates in this paper are evaluated at the *robust verification point*

$$(\rho_0, e, a, \tau_0, L_z) = (0.5 \text{ m}, 0.2, 0.02, -1 \text{ m}^2, 1),$$

found by a maximin search after the first exhibited point's tolerance claim failed on a finer grid. The ten knobs are: the collar width w ; the blend widths d_C, d_F, d_L of the tilt, F - and L -levers; f_{conc} , the concavity/bound parameter of the F -lever; l_{conv} , the L -growth amplitude; and (s_d, s_w, t_d, t_w) , the start offset and width of the tracefree and trace K -relaxation bands. The vector below is $(w, d_C, d_F, d_L, f_{\text{conc}}, l_{\text{conv}}, s_d, s_w, t_d, t_w)$; its explicit profiles are in Appendix A.

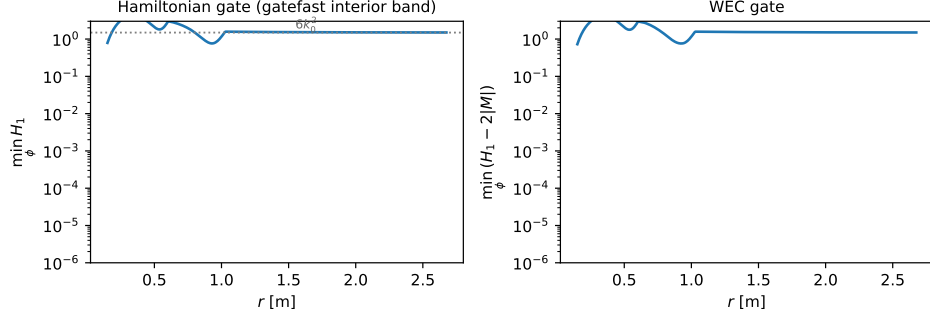


Figure 1: Both gates over the gatefast interior band $r \geq w + 0.0904$ of the C4 layer at the retuned knob point, $a = 0.02$ (fine grid: $\Delta r = 0.005$, 48ϕ ; minima over ϕ). Band minima: $+0.763129$ and $+0.728189$; the dotted line is $6k_0^2$, the RW density the envelope asymptotes to. The innermost collar, $r < w + 0.0904$, is not shown; at the retuned point it is gate-positive (exact-jet certificate: $WEC(1\text{ mm}) \geq +4.55 \times 10^{-6}$ at $a = 0.02$; dense scan positive on $r > w$), with the exact vacuum boundary $r = w$ marginal ($H_1 = 0$ exactly).

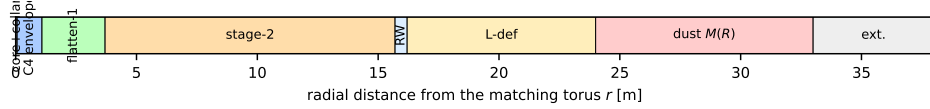


Figure 2: The composite chain, radially from the matching torus: core/collar, C4 envelope, two flattening stages (the second closes the fat-cone gap), then Ori's RW, L -deformation, dust and exterior layers (re-verified).

At the retuned knob point

$$(w, \dots, t_w) = \begin{pmatrix} 0.0601 & 1.029 & 0.5424 & 0.9675 & 0.8498 \\ 0.0932 & 0.0015 & 0.4003 & 0.1973 & 0.6801 \end{pmatrix} \quad (6)$$

(s_d is the only knob changed from the previously exhibited point: $0 \rightarrow 0.0015$, a 1.5 mm delay of the stage-1 onset; Sec. 4) on the gatefast interior band (fine grid):

$$\min H_1 = +0.763129, \quad \min (H_1 - 2|M|) = +0.728189 \quad (a = 0.02), \quad (7)$$

with $+0.799072$ and $+0.771047$ at $a = 0$. These are interior-band sampled margins. The retuned construction is also gate-positive on the full envelope

layer $r > w$, including the innermost collar: the stage-1 K -relaxation onset is delayed by $s_d = 1.5$ mm, so $K = K_c$ exactly for $r < w + s_d$ and the departure has lifted H_1 before the onset acts. At $a = 0.02$ the exact interval jets certify $WEC(1\text{ mm}) \geq +4.550484 \times 10^{-6}$, and a dense exact-jet scan over $r > w$ is positive everywhere ($\min H_1 = \min WEC = +2.4 \times 10^{-12}$ at the innermost sampled shell); the boundary torus $r = w$ is the exact vacuum continuation ($H_1 = 0$ exactly), as it must be. The previous 1.5 mm collar marginality is removed. Robustness: all 21 configurations (nominal + every single knob at $\pm 30\%$) pass both gates on the same band (worst margin $+0.39$). The 21/21 set is the stated robustness check; per-knob tolerance widths were not re-measured after the collar retune. Uniformity in the core parameter: at fixed knobs, $a \in \{0.02, 0.005, 0.001, 0\}$ gives interior-band WEC margins $+0.7282 \rightarrow +0.7710$ (Hamiltonian margins $+0.7631 \rightarrow +0.7991$), monotonically to the exact Misner limit [4] — the numerical content of the $O(a)$ stability conjecture. *Interval-certified*: on the post-relaxation region $r \in [1.0891, 2.68]$ (where the K -bands are complete and $C = 0$, so $K = k_0 h$), interval evaluation of the warped-product Ricci (2) over all cells gives $|R^{(3)}| \leq 0.1559$, hence $\min H_1 > +1.336$ for every $a \in [0, 0.02]$.

Conjecture ($O(a)$ stability). For every $a \in [0, a_0]$ the exhibited profile family (knobs independent of a) satisfies both gates, with margins continuous and monotone in a at $a = 0$. Exhibited: $a \in \{0.02, 0.005, 0.001, 0\}$, WEC margins $+0.7282 \rightarrow +0.7710$ (Hamiltonian margins $+0.7631 \rightarrow +0.7991$); the boundary inputs are the shape residual $\sigma = 0.5247 a$ at $\tau_0 = -1$ ($\approx 1.08 a/2|\tau_0|$ there, with the $a/\sqrt{|\tau_0|}$ depth trend noted above) and $\Delta(R^{(3)} + 6k_0^2) = \pm 1.25 a \rho^2 / \tau_0^2$.

Restricted result (interval-certified). On the gatefast band $r \geq w + 0.0904$ an interval enclosure of the exact analytic jets gives, for every $a \in [0, 0.02]$,

$$H_1(a) \geq 0.7468, \quad H_1(a) - 2|M|(a) \geq 0.2736,$$

with certified derivative bounds

$$\sup |dH_1/da| \leq 4.106, \quad \sup |d(H_1 - 2|M|)/da| \leq 12.754,$$

and certified slopes $m_H(0) \geq 0.798725$, $m_W(0) \geq 0.441002$; the Lipschitz chain gives $H_1 \geq 0.7166$ and $WEC \geq 0.1859$ at $a = 0.02$, so both gates hold there and $a_W^* = 0.0346$ (certified, still > 0.02). The retuned collar is gate-positive at $a = 0.02$ (Sec. 4): the exact point certificate gives $WEC(1\text{ mm}) \geq +4.550484 \times 10^{-6}$, the dense exact-jet scan over $r > w$ is positive, and the exact vacuum boundary $r = w$ remains marginal ($H_1 = 0$ exactly).

Independently, the interval $R^{(3)}$ bound gives $H_1 > 1.336$ on $[1.0891, 2.68]$ for every $a \in [0, 0.02]$ (Appendix A). The full fixed-knob open-band uniformity for all $a \leq a_0$ remains a conjecture; the flat-band elliptic constant is proved ($C_\ell = \sqrt{2}$), while the composite-background constant is conditional.

5 The hand-off and Ori's outer layers

Ori's layers 2–3, the RW layer, the spherical dust shell and the exterior attach only to his Layer-2 form $ds^2 = dr^2 + F^2 d\phi^2 + L_0^2 dz^2$ (F linear, L_0 constant, $K = k_0 h$) and are core-independent. *Stage 1* drives the C4 outer edge onto that form exactly: $|F''|, |\partial_r L|, |L(\phi) - \bar{L}|, |C|, |K - k_0 h| \leq 2 \times 10^{-6}$ (FD noise), and $H_1(R_{2P}) = 1.494007$ vs $6k_0^2 = 1.494006$: the RW density ε_0 to five decimals. *Stage 2* (added in closing a real gap): the stage-1 state is a *fat cone* — $F' = 0.79 \neq 1$ and $F = 3.52 > \bar{L} = 1.13$ — to which Ori's L -deformation cannot attach (his eqs. (21–22) need a unit-slope base and $L_0 > r_0 \theta_3$). A second band bends $F' \rightarrow 1$ and grows $L \rightarrow L^* = 20$; both gates hold ($\min H_1 = +0.353$) and his bound is met.

We then *re-implement and re-verify all of Ori's outer layers*: the RW layer ($H_1 = 6k_0^2$ to 10^{-16}), the L -deformation, the spherical dust layer with mass function $M(R)$, and the exterior. The load-bearing anchor — our machinery reproducing his closed form $R^{(3)} = -2/(9\mu^2) + 4M'/R^2$ — holds to 3.5×10^{-8} ; his eqs. (18), (20), (24)–(25) and the $\Delta L \leq L$ bound reproduce with residual zero; the μ - k_0 bookkeeping ($2/9\mu^2 = 6k_0^2$) is exact to 2×10^{-16} . Every inter-region join is C^2 (metric mismatch zero exactly; first derivatives $\leq 2 \times 10^{-8}$) with constraints continuous to 4×10^{-6} : no thin shells anywhere. Ori's exterior layer is asymptotically flat by construction (his M2 is an exterior Schwarzschild slice); we rely on that for asymptotic flatness and do not independently compute the ADM mass or momenta. The outer composite chain is verified end to end; the canonical family's deep-rung extrapolation and the open-band $O(a)$ conjecture are the two stated exceptions (Secs. 6 and 4).

6 The scaling group and the mass dial

The scaling group is a chart-dilation congruence: with $\Lambda = \text{diag}(\lambda, 1, 1)$ and $\Lambda^T h_\lambda(\lambda r) \Lambda = \lambda^2 h$, the map

$$(\rho_0, \tau_0, \ell_i) \rightarrow (\lambda \rho_0, \lambda^2 \tau_0, \lambda \ell_i) \quad \text{at fixed } (a, e, L_z) \quad (8)$$

takes solutions to solutions, where the length knobs ℓ_i are the profile lengths ($d_C, d_F, d_L, w, \dots$), not the z -period. The gates scale as $1/\lambda^2$; $\lambda = 2$ is confirmed to 8.4×10^{-9} .

Mass: the dust *density* scales as $\varepsilon_0 = 6k_0^2/16\pi$, but at fixed ρ_0 and dimensionless z -period L_z the total mass does *not* scale as k_0^2 : the z -orbit proper length grows as $1/k_0$, so the volume does too, and the certified integral gives

$$M_{\text{layer}} = 3.30 |k_0| L_z \quad (9)$$

in geometrized units, per unit dimensionless z -period. For the C4 layer this is exactly linear in $|k_0|$, with a coefficient certified numerically across four rungs; the physical conversion is $M_{\text{phys}} = (c^2/G)M_{\text{geo}}$ with $c^2/G = 1.347 \times 10^{27} \text{ kg/m}$. Here M_{layer} is the C4 layer (open layer through the flattening band). At the deep rung $|k_0| = 2.2 \times 10^{-12}$ ($\tau_0 \simeq -5 \times 10^{22} \text{ m}^2$) this extrapolates to $9.8 \times 10^{15} \text{ kg} \cdot L_z$, at an ordinary density ($\varepsilon_0 = 778 \text{ kg/m}^3$; mean ~ 850 , peak $\sim 2300 \text{ kg/m}^3$). Adding the RW tube gives $1.3\text{--}2.4 \times 10^{16} \text{ kg} \cdot L_z$ (lower/upper envelope bounds; the upper includes Ori's outer layer, and the range is not an error bar). The $|k_0|$ -linear law is demonstrated for the C4 layer and the RW tube; we have not established it for the full upper envelope that includes Ori's outer layer.

The canonical corner is $|k_0| = 0.499$ ($\tau_0 = -1 \text{ m}^2$; the signed trace convention used by the module is $k_0 = -1/(2\sqrt{|\tau_0|}) = -0.4990$). There $\varepsilon_0 = 4.0 \times 10^{25} \text{ kg/m}^3$ (peak $1.17 \times 10^{26} \text{ kg/m}^3$, $\sim 5 \times 10^8$ times nuclear saturation): not ordinary matter. The RW tube alone is $M_{\text{tube}} = \varepsilon_0 2\pi \bar{L} [F_2 d + F_2' d^2/2] = 6.45 \times 10^{26} \text{ kg}$; the full canonical envelope is $2.9\text{--}5.4 \times 10^{27} \text{ kg}$. The tube is a component, not the machine mass.

Provenance. The authoritative computation is `massdial_verify.py` in `simulations/` (40-digit Laurent q -series, second-order jets, fine grid), written up in the accompanying technical note. The present-day engineering extraction, `wrap_engineering.py`, has been updated to this corrected law (masses $\propto |k_0|$, densities $\propto k_0^2$). The per-depth retuning is stated honestly: the exhibited family is uniform in a at fixed design depth, but it is not uniform in τ_0 without retuning; the mass-dial check exhibits a single-knob depth-uniform retune, not a general proof.

7 Claims ledger

CERTIFIED (interval arithmetic): post-relaxation region $[1.0891, 2.68]$, $\min H_1 > 1.336$ from $|R^{(3)}| \leq 0.1559$. Mass law: density $\propto k_0^2$ and $M_{\text{layer}} = 3.30 |k_0| L_z$ at fixed (ρ_0, L_z) for the C4 layer — exact linear scaling, with

a coefficient certified numerically at four rungs (`massdial_verify.py`, in `simulations/`). The four-rung evaluation, including the tons rung, is for the separately retuned deep family used by that script, not for the canonical knob set.

EXTRAPOLATED the canonical family’s ordinary-density deep-rung point ($|k_0| \simeq 2.2 \times 10^{-12}$): its density and mass follow by extrapolating the certified k_0^2 density and $|k_0|$ -linear mass laws; it is not directly gate-evaluated at depth.

EXACT (closed form/sympy): collar vacuum; slice family; scaling group (the chart-dilation part); Ori’s layer formulas (re-derived).

EXHIBITED (fine-grid, machinery validated to 10^{-16}): gatefast interior-band gates; robustness 21/21; uniformity in a to the Misner limit (four values, margins monotone); hand-off; stage-2; join continuity. At the retuned point ($s_d = 0.0015$) both gates are positive over the full envelope layer $r > w$ including the innermost collar at $a = 0.02$ (exact-jet point certificate $WEC(1\text{ mm}) \geq +4.550484 \times 10^{-6}$ and a dense positive scan for $r > w$); the exact vacuum boundary $r = w$ is marginal ($H_1 = 0$).

INTERVAL-CERTIFIED (RESTRICTED) $O(a)$ stability on the gatefast band $r \geq w + 0.0904$: both gates hold for every $a \in [0, 0.02]$ with certified direct bounds $H_1 \geq 0.7468$, $WEC \geq 0.2736$, certified derivative bounds $\sup |dH_1/da| \leq 4.106$, $\sup |dWEC/da| \leq 12.754$, and $a_W^* = 0.0346$; $H_1 > 1.336$ is interval-certified on $[1.0891, 2.68]$ (Appendix A). The flat-band elliptic constant $C_\ell = \sqrt{2}$ is proved; the composite-background constant remains conditional. An $O(a)$ -corrected family exists under that conditional estimate (not proved).

CONJECTURE $O(a)$ stability: the exhibited knob family satisfies both gates on the full open band for every $a \in [0, a_0]$ (uniformity exhibited; the interval-certified restricted result above covers the gatefast band; no analytic proof of the full statement).

CITED (Ori 2005, 2007): the evolution existence theorem (assuming its hypotheses apply to the composite data; we have not re-proved it); the outer-layer construction in his parameters (re-verified here). The chronology-protection context [3, 4, 5] and the classical no-go results [6, 7, 8, 9, 10] constrain the assembly question; the switchable-family direction is Xavier’s [11].

OPEN a certified continuum (cell) bound over the collar (the point and dense exact-jet certificates are in hand, and no negative sign remains to confine); the composite-background elliptic constant; analytic error bounds on the remaining band region; a proof of the $O(a)$ conjecture.

SCOPE / NOT ADDRESSED late-time evolution; assembly/acquisition of the identified core (no known classical process; the obstruction literature is cited in Sec. 8); per-depth retuning; analytic error bounds.

8 Discussion

The composite initial data — vacuum identified core, exact collar, robust dust envelope, verified hand-off, re-verified outer layers — exists explicitly and satisfies every classical requirement on the gatefast interior band: regular initial data, asymptotically flat through Ori’s exterior (we do not independently compute the ADM charges), non-exotic dust (at ordinary density only at the deep rung; for the canonical family that point follows by extrapolating the certified scaling law, while a separately retuned deep family is gate-evaluated there), with both gates positive and interval-certified margins on the interior band and, at the retuned point, over the full envelope layer $r > w$ including the innermost collar. In Ori’s own framing the time machine *is* the evolved spacetime of this data; assuming his theorem applies to the composite data — we have not re-proved it — the evolution produces a chronology horizon, where the lapse vanishes, and closed timelike curves. What we have added: the transplant (C3, C4), its robustness and certification, the mass dial (density $\propto k_0^2$ but C4-layer mass $\propto |k_0|$ at fixed ρ_0 and L_z ; the canonical RW tube is 6.45×10^{26} kg and the full canonical envelope is $2.9\text{--}5.4 \times 10^{27}$ kg), and an end-to-end verification of the outer construction chain, with the canonical family’s deep-rung extrapolation and the open-band $O(a)$ uniformity as the two stated exceptions; late-time evolution is not analyzed.

What remains is in the ledger: a certified continuum (cell) bound over the collar, the composite-background elliptic constant, analytic bounds for the remaining band region, a proof of the $O(a)$ conjecture (uniformity exhibited), and the per-depth problem. Outside the existence claim, the boost identification itself has no known classical manufacturing process: the data must be acquired, not assembled. We state this as an open no-go conjecture, not a theorem. The supporting evidence is the classical no-go literature on producing such data from controlled initial data: the constant-mean-curvature obstruction of Carroll, Farhi and Guth [6] and its Olum et al.

extension [7], the singular initial values of Deser, Jackiw and 't Hooft [8] and Cutler [9] for Gott's strings, and topological censorship [10]. These results do not cover the present construction, which starts from the identified core; they constrain attempts to assemble it. That distinction, and the survey for natural identified regions, is the program's next stage.

A Explicit profiles and numerical methods

Chart and data. On the z -invariant chart (r, ϕ, z) of Sec. 4, let $\rho = \rho_0 + r$, with r the outward distance from the matching torus T_0 . The data are

$$h = \begin{pmatrix} 1 & 0 & C \\ 0 & F^2 & 0 \\ C & 0 & L^2 \end{pmatrix}, \quad K = \text{the exact core } K \text{ blended to } k_0 h, \quad (10)$$

with $C = C(r)$, $F = F(r)$, $L = L(r, \phi)$.

Collar and lift levers. For $r \leq w$ the data are the exact vacuum continuation ($H_1 = M^i = 0$ to machine precision). For $r > w$, write $u_X = (r - w)/d_X$ and, clipped to $[0, 1]$,

$$S_3(u) = u^3(10 - 15u + 6u^2), \quad S_7(u) = u^4(35 - 84u + 70u^2 - 20u^3). \quad (11)$$

S_3 is the C^2 smoothstep; S_7 is the quartic-onset C^3 blend used for the one profile with a mismatch at w . The levers are

$$C(r) = \frac{\rho}{2} [1 - S_7(u_C)], \quad (12)$$

$$F(r) = \rho [1 - S_3(u_F)] + F_t S_3(u_F), \quad (13)$$

$$F_t = (\rho_0 + w) + r_F - \frac{f_{\text{conc}} r_F^2}{2(1 + r_F/\lambda)}, \quad (14)$$

$$L(r, \phi) = L_b [1 - S_3(u_L)] + L_t S_3(u_L), \quad (15)$$

$$L_t = \mathcal{A} \left[1 + l_{\text{conv}} \left(1 - e^{-r_L/0.8} \right) \right], \quad (16)$$

with $\lambda = 0.5$, $r_F = r_L = r - w$, and the derivative match $L'_t(w) = L'_b(w)$. Here L_b and $\mathcal{A}$ are the base and collar-edge values fixed by the umbilical-slice metric,

$$L_b = \sqrt{\frac{\rho^2}{4} - \tau_0 + \frac{a}{2}\rho^2 \cos 2\phi}, \quad (17)$$

$$\mathcal{A} = \sqrt{\frac{(\rho_0 + w)^2}{4} - \tau_0 + \frac{a}{2}(\rho_0 + w)^2 \cos 2\phi}. \quad (18)$$

K -relaxation. With K_c the core extrinsic curvature and

$$k_c = \text{tr}(h^{-1}K_c)/3,$$

the stages are

$$K_1 = (1 - u_1)K_c + u_1(k_ch), \quad u_1 = S_3\left(\frac{r - w - s_d}{s_w}\right), \quad (19)$$

$$K = (1 - u_2)K_1 + u_2(k_0h), \quad u_2 = S_3\left(\frac{r - w - t_d}{t_w}\right), \quad (20)$$

so stage 1 relaxes the tracefree part and stage 2 the trace. At the retuned point the bands are approximately $[0.062, 0.462]$ and $[0.257, 0.938]$; $K = k_0h$ is complete by $r \simeq 0.938$, but the C , F and L blends complete at $r = w + d_C = 1.0891$, where the certified post-relaxation region begins.

Flattening to Ori's Layer 2. Stage 1 ($r \in [R_1, R_{2P}] = [2.7, 3.7]$) blends F onto its tangent line,

$$F = (1 - S_3)F_{C4} + S_3F_{\text{lin}}, \quad F_{\text{lin}} = F(R_1) + F'(R_1)(r - R_1), \quad (21)$$

with $S_3 = S_3((r - R_1)/1.0)$, sets $L \rightarrow \bar{L}(R_1)$, and leaves $C = 0$. At R_{2P} the residuals are $|F''|, |\partial_r L|, |L - \bar{L}|, |C|, |K - k_0h| \leq 2 \times 10^{-6}$ and $H_1(R_{2P}) = 1.494007$ versus $6k_0^2 = 1.494006$. Stage 2 ($[R_{2P}, R_{2P} + 12]$) bends $F' \rightarrow 1$ and $L \rightarrow L^* = 20$, meeting Ori's unit-slope and $L_0 > r_0\theta_3$ conditions; both gates hold with $\min H_1 = +0.353$.

Gate evaluation and certification. Every C4 gate claim is evaluated on the gatefast interior band $r \in [w + 0.0904, R_1]$ on the fixed fine grid $\Delta r = 0.005$, 48 azimuths (`gatefast.py`, in `simulations/`), with noise floor 10^{-7} ; the flattening bands are evaluated on their own fine grids. The collar, where the gates are $O((r - w)^2)$ and below the finite-difference scale of the momentum stencil, is evaluated with the exact analytic jets instead. The einsum-vectorized Ricci and momentum machinery agrees with an independent scalar implementation to 10^{-16} at random points. On the post-relaxation region $r \in [1.0891, 2.68]$, where $K = k_0h$ and $C = 0$, the warped-product Ricci (2) is evaluated in interval arithmetic over subdivided cells, giving $|R^{(3)}| \leq 0.1559$ and hence $\min H_1 > +1.336$ for every $a \in [0, 0.02]$. The mass coefficient is computed by `massdial_verify.py` (in `simulations/`). It uses exact Laurent series in $q = k_0$ through q^{10} with 40-digit `mpmath`, second-order jets for the profile derivatives, on the same $\Delta r = 0.005$, 48 ϕ

grid; it evaluates the series at four rungs, including the tons rung, and certifies the q^2 density scaling and the linear mass coefficient there for the separately retuned deep family it uses ($d_C = 1.7$, $a = 10^{-3}$). It does not gate-evaluate the canonical $a = 0.02$, $d_C = 1.029$ family at depth; that family's deep-rung point is the scaling extrapolation recorded in the ledger.

$O(a)$ error bounds. On the relaxation-complete region $r \geq w + d_C = 1.0891$ the data reduce to $F = F_t$, $L = \mathcal{A}g(r)$, $K = k_0 h$; the triangle bound

$$|R^{(3)}| \leq 2 \left[\frac{|g''|}{g} + \frac{|F''|}{F} + \frac{|F'| |g'|}{Fg} + \frac{2\varepsilon}{(1-\varepsilon)F^2 g} \right], \quad \varepsilon = \frac{aB_c}{A}, \quad (22)$$

with $B_c = (\rho_0 + w)^2/2$ and $A = (\rho_0 + w)^2/4 + |\tau_0|$, gives $|R^{(3)}| \leq 0.1559$ for every $a \in [0, 0.02]$ on $[1.0891, 2.68]$, hence $H_1 > 1.336$. The a -dependence there is the exact frozen-quadrupole law

$$R^{(3)}(a) - R^{(3)}(0) = \frac{2}{F^2} \frac{2\varepsilon \cos 2\phi + \varepsilon^2(1 + \cos^2 2\phi)}{(1 + \varepsilon \cos 2\phi)^2} \leq \frac{4\varepsilon}{F^2(1 - \varepsilon)}. \quad (23)$$

The interval-certified jets give $m_H(0) \geq 0.798725$, $m_W(0) \geq 0.441002$, $\sup |dH_1/da| \leq 4.106$, $\sup |dWEC/da| \leq 12.754$, and the direct value certificate $H_1 \geq 0.7468$, $WEC \geq 0.2736$; the Lipschitz chain gives

$$H_1(a) \geq 0.798725 - 4.106 a, \quad WEC(a) \geq 0.441002 - 12.754 a, \quad (24)$$

so $a_W^* = 0.0346$ (certified, still > 0.02) and both gates hold on $r \geq w + 0.0904$ for every $a \in [0, 0.02]$. At the retuned point ($s_d = 1.5$ mm; this is the only changed knob) the exact jets certify $WEC(1 \text{ mm}) \geq +4.550484 \times 10^{-6}$ at $a = 0.02$, and the dense scan over the collar is positive for $r > w$; the exact vacuum boundary $r = w$ is marginal. The flat-band elliptic constant is proved ($C_\ell = \sqrt{2}$); the composite-background constant is the remaining open estimate. The lever bounds used are

$$\begin{aligned} |C| &\leq 0.3811, & |C_r| &\leq 0.9737, & |C_{rr}| &\leq 4.4455, \\ F &\geq 0.5601, & F' &\geq 0.7920, & |F''| &\leq 0.9841, \\ L &\geq 1.0370, & |L_r| &\leq 0.1695, & |L_{rr}| &\leq 0.6242, & |L_{,\phi\phi}| &\leq 0.0131, \end{aligned}$$

with the full derivation in this appendix and the validation record in the project repository.

Reproducibility. The script `simulations/hybrid_oa_bounds.py` reproduces the $O(a)$ bounds. The `--quick` run takes ~ 50 s, the full run ~ 2 min:

```
.venv/bin/python simulations/hybrid_oa_bounds.py --quick  
.venv/bin/python simulations/hybrid_oa_bounds.py
```

The `--json` flag writes a small result table under `data/hybrid_oa/` (git-ignored). The script imports the existing envelope and gate machinery, modifies no profile, and uses no network.

References

- [1] A. Ori, “A class of time-machine solutions with a compact vacuum core,” *Phys. Rev. Lett.* **95**, 021101 (2005), gr-qc/0503077.
- [2] A. Ori, “Formation of closed timelike curves in a composite vacuum/dust asymptotically-flat spacetime,” *Phys. Rev. D* **76**, 044002 (2007), gr-qc/0701024.
- [3] S. W. Hawking, “Chronology protection conjecture,” *Phys. Rev. D* **46**, 603 (1992).
- [4] C. W. Misner, “Taub-NUT space as a counterexample to almost anything,” in *Relativity Theory and Astrophysics I: Relativity and Cosmology*, Lectures in Applied Mathematics Vol. 8, edited by J. Ehlers (American Mathematical Society, Providence, 1967), pp. 160–169.
- [5] B. S. Kay, M. Radzikowski, R. M. Wald, “Quantum field theory on spacetimes with a compactly generated Cauchy horizon,” *Commun. Math. Phys.* **183**, 533 (1997).
- [6] S. M. Carroll, E. Farhi, A. H. Guth, “An obstacle to building a time machine,” *Phys. Rev. Lett.* **68**, 263 (1992).
- [7] S. M. Carroll, E. Farhi, A. H. Guth, K. D. Olum, “Energy-momentum restrictions on the creation of Gott time machines,” *Phys. Rev. D* **50**, 6190 (1994), gr-qc/9404065.
- [8] S. Deser, R. Jackiw, G. ’t Hooft, “Physical cosmic strings do not generate closed timelike curves,” *Phys. Rev. Lett.* **68**, 267 (1992).
- [9] C. Cutler, “Global structure of Gott’s two-string spacetime,” *Phys. Rev. D* **45**, 487 (1992).

- [10] J. L. Friedman, K. Schleich, D. M. Witt, “Topological censorship,”
Phys. Rev. Lett. **71**, 1486 (1993).
- [11] S. Xavier, “Closed timelike curves from a vacuum traveling wave,”
arXiv:2607.00788 (2026).